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Vectors, pure and applied. A general introduction to linear algebra.

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This book is a general introduction to linear algebra. According to the author, the objective of this book is to help future mathematicians to see the vectors from the point of view of algebra, analysis, physics or numerical analysis. It is based on those parts of the first and second year Cambridge courses which deal with vectors. The book is divided into two parts, corresponding to the first and second year material, respectively. The concepts reappear in increasingly general forms. For example, in the first part, the inner product starts as a tool in two or three dimensional geometry and is then extended to $\mathbb{R}^n$ and $\mathbb{C}^n$. In the second part, it reappears as an object satisfying certain axioms. The material presented in this book has been distributed in the following chapters: Part I: Familiar vector spaces ; 1. Gaussian elimination; 2. A little geometry; 3. The algebra of square matrices; 4. The secret life of determinants; 5. Abstract vector spaces; 6. Linear maps from $\mathbb{F}^n$ to itself; 7. Distance preserving linear maps; 8. Diagonalisation for orthonormal bases; 9. Cartesian tensors; 10. More on tensors; Part II: General vector spaces; 11. Spaces of linear maps; 12. Polynomials in $\mathcal{L}(U,U)$; 13. Vector spaces without distances; 14. Vector spaces with distances; 15. More distances; 16. Quadratic forms and their relatives. Probably the most important problem in mathematics is that of solving a system of linear equations. This book begins with a chapter on linear systems, describing the Gaussian elimination and introducing the matrix notation. The column vectors introduced in the first chapter are used in Chapter 2 to study some aspects of geometry. Beginning in $\mathbb{R}^2$ and $\mathbb{R}^3$, the author analyzes some concepts such as the straight line joining two points, the Euclidean distance, the angle between two vectors, ... In Chapter 3, the author defines arithmetic operations with square matrices and looks at some of their algebraic properties. Elementary matrices are introduced and the LU factorization and the Jordan-Gauss algorithm for the calculus of the inverse are described. With each square matrix it is possible to associate a real number called the determinant of the matrix. The author presents this concept in Chapter 4, studies its properties and derives an elimination method for evaluating determinants. The operations of addition and scalar multiplication are used in many diverse contexts in mathematics. Thus a general theory of mathematical systems involving these operations is developed in Chapter 5. The notions of vector space, basis, dimension, linear map,... are some of the concepts introduced in this chapter. There is a close relationship between linear mappings, between finite dimensional spaces, and matrices. This relationship is discussed in Chapter 6. The notion of orthogonality in $\mathbb{R}^n$ and related properties is introduced in Chapter 7. Reflection in $\mathbb{R}^n$, $n \geq 2$, QR factorization and the Householder transformation are the topics developed by the author. With this concept, he addresses the diagonalization problem of linear mappings or matrices with respect to some orthonormal basis. Part I finishes with two chapters devoted to develop the idea of a Cartesian tensor. In the first part of this book, the n-dimensional vector spaces are seen as generalizations of two- and three-dimensional spaces. In the second part, the reader should look at n-dimensional vector spaces with an eye to generalization to infinite dimensional spaces. In Chapter 11, the author studies the special vector space $\mathcal{L}(U,U)$, that is, the vector space of the linear mappings of a vector space U into itself. He also analyzes the dual space of U using and without using bases. Chapter 12 deals with polynomials in $\mathcal{L}(U,U)$. The concepts of direct sum and minimal polynomial allow to obtain necessary and sufficient conditions for the diagonalizability of a linear mapping. In the other case, the Jordan form of the linear mapping can be obtained. There are at least two ways how the notion of a finite dimensional vector space over $\mathbb{R}$ or $\mathbb{C}$ can be generalized. The first is that of the analyst who considers infinite dimensional spaces. The second is that of the algebraist who considers finite dimensional vector spaces over more general objects than $\mathbb{R}$ or $\mathbb{C}$. Chapter 13 addresses this second point of view. Moreover, Chapters 14 and 15 are devoted to vector spaces with distances. The general concept of real (complex) inner product space is introduced, the case $\mathcal{C}([a,b])$ is studied and the relationship between inner product and dual space is analyzed. Other inner product spaces are described in Chapter 15. Finally, Chapter 16 is devoted to bilinear and quadratic forms and their relatives. Throughout the book there are two sorts of exercises. The first form is part of the text and provides the reader with an opportunity to think about what have just been done. The second type occurs at the end of each chapter and some of them provide extra background.

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Classification: H65

Keywords: vector space; linear map; matrix; Euclidean distance; inner product; QR factorization; diagonalization; Jordan form; bilinear form; textbook; matrix inversion; Gaussian elimination; determinants; Cartesian tensors; quadratic forms; system of linear equations; LU factorization; Jordan-Gauss algorithm;

Householder transformation; minimal polynomial; inner product space; quadratic form
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