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Guin, Daniel

Algebra. Volume 2. Rings, modules and multilinear algebra. L3, M1, M2. (Algèbre. Tome 2. Anneaux, modules et algèbre multilinéaire. L3, M1, M2.)

Collection Enseignement SUP. Mathématiques. Les Ulis: EDP Sciences (ISBN 978-2-7598-1001-7/pbk). xiv, 244 p. (2013).

The volume under review is the sequel to the French algebra textbook "Algèbre I. Groupes, corps et théorie de Galois" [Les Ulis: EDP Sciences. (2008; Zbl 1153.12001)] by the author and *T. Hausberger*. Basically, the entire two-volume text is geared toward upper-level undergraduates and graduate students in mathematics at French universities. Also, it is meant to serve as a profound source for those candidates preparing for the major civil-service examinations CAPES or "L'agrégation" in France. While the first volume provides a comprehensive introduction to the elements of group theory, field theory and Galois theory, respectively, the current second volume, this time with the author as the only author, is devoted to the complementary topics of commutative ring theory, module theory, and multilinear algebra. Accordingly, the material is organized in two principal parts consisting of several chapters each. Within this didactic disposition, Part I comprises the first seven chapters covering the following topics successively: Chapter 1 gives an introduction to the fundamental concepts of commutative ring theory such as homomorphisms, ideals, prime and maximal ideals, products of rings and the Chinese remainder theorem, the characteristic of a ring, and quotient fields of integral domains. Chapter 2 discusses polynomial rings, Euclidean rings, principal ideal domains, factorial rings, and the notion of divisibility in rings. Chapter 3 continues the study of polynomial rings, with particular emphasis on irreducible polynomials, resultants and discriminants, derivations, and symmetric polynomials. Chapter 4 turns to modules over commutative rings and their most general properties, including direct sums and products of modules as well as free modules and algebras in the course of the discussion, the structure theorem for finitely generated modules over a principal ideal theorem is the main topic of Chapter 5, while the subsequent Chapter 6 deals with some arithmetic concepts in ring theory. More precisely, this chapter studies integral ring extensions, the notions of norm and trace, Noetherian rings and modules, fractional ideals, Dedekind rings, the norm of an ideal, the decomposition of prime ideals in ring extensions, and the rudiments of ramification theory in this context. Chapter 7 is devoted to the properties of the Hom-functor in module theory and the related duality theory, with particular emphasis on finitely generated free modules. Each of the Chapters 1–7 concludes with a section titled "Thèmes de réflexion", where additional topics are introduced through carefully guided exercises. Among the themes inviting the reader to independent work are related topics such as power series, Laurent series, further irreducibility criteria for polynomials, universal properties of special module constructions, the Jordan nomal form of a vector space endomorphism, prime numbers and their ramification in number fields, injective and projective modules, the infective hull, and modules of finite length. Part II of the present book is titled "Multilinear algebra" and comprises the remaining two chapters. Chapter 8 introduces tensor products of modules and associative algebras, together with their basic (universal) properties. The exercises to this chapter provide some basic facts on flat and faithfully flat modules, respectively. Chapter 9 finally treats alternating multilinear maps of modules, determinants of square matrices over a ring, the exterior products of a finitely generated free module, and the Grassmann algebra. The exercises acquaint the reader with such related topics like derivations on a ring with values in a module, algebraic differential forms, and vanishing properties of exterior powers. The main text of the book is enhanced by two appendices providing some general tools from set theory as used in the course of the treatise. These appendices concern orderings and well-orderings on sets, on the one hand, and cardinalities of infinite sets on the other. Overall, the presentation of the material is characterized by a high degree of clarity, rigor, and expository skill. The wealth of accompanying exercises, illustrating examples, and instructive remarks is another feature of this excellent textbook which, besides, appears to be largely self-contained and pleasantly versatile. *Werner Kleinert (Berlin)*

Classification: H45 H65 H75
Keywords: commutative rings; ideals; factorial rings; polynomial rings; localization; modules; Dedekind rings; tensor algebra; exterior algebra; derivations; differential forms