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**Distilling ideas. An introduction to mathematical thinking: graphs, groups, calculus.**

MAA Textbooks. Washington, DC: The Mathematical Association of America (MAA) (ISBN 978-1-939512-03-1/pbk; 978-1-61444-613-2/ebook). xvi, 171 p. (2013).

This well written and enjoyable text introduces the reader to three commonly studied disciplines of pure mathematics: graph theory, group theory and calculus. The style is conversational but all definitions and theorems are still rigorous in their statement. The authors approach these topics using inquiry-based learning; each broad topic is introduced with a number of real-life examples and the similarity between these examples and essential foundations of the topic are distilled out through a process of inquiry. It is the reader who is expected to answer a series of questions uncovering the foundational definitions and theories of the discipline. The presentation of examples is often lighthearted, to the point of humorous, which only helps the student become involved in the subject material. The reviewer particularly liked the adventures of Zeno, Gottfried and Isaac used to introduce calculus and limits. The book is a suitable alternative to standard texts on graph or group theory or calculus particularly if the learning is focused on introducing the concept of proof or on covering a broad range of pure mathematical concepts. It is a very suitable resource for training school teachers or scientists in the basics of mathematical thinking. Although the book does not go far enough in any one of the topics to be considered as a text for a pure mathematics course, the techniques and approaches to understanding these topics could be beneficially used within a standard course. The inquiry-based learning requires some assistance from an instructor; before this text is suitable for independent study it needs a section of solutions to the exercises and proofs to the theorems, some of which are too challenging, without specific hints, for a novice (for example the Euler circuit theorem 2.31). Without an instructor, the reader could not be certain that their inquiry had resulted in correctly answering the exercises and performing proofs. The topic of graphs starts with the Königsberg bridge problem, a paperperson's delivery route and a network of gas, water and electricity connections. The investigation following abstracts these situations to sets of vertices and edges and from here to all the standard introductory theory of graphs including walks, circuits, connectivity, planarity, Euler characyeristic, colourability of maps, dual graphs and colouring and the five colour theorem. There is a discussion of the four colour problem and use of computers in it's proof. The chapter is wrapped up with a stocktake of the techniques of inquiry used to distill out the fundamental concepts of graph theory. The topic of groups begins with the examples of integers with addition, non-zero real numbers and multiplication and the symmetry group of a triangle. The authors unfold such concepts as associativity, commutativity, uniqueness of identity and inverse, cyclic groups, Cayley tables, general dihedral groups, subgroups, generators, group centre. The authors take the reader further exploring cosets, Lagrange's theorem, permutation groups, symmetric group of bijections on a set $X$, homomorphisms, kernels and images, conjugation, equivalence relations, normal subgroups, abelianization and the action of a group on a set. The chapter ends, again, with a summary of the techniques used to explore group theory. The chapter on calculus begins at the Olympic games with a problem of convergence of an arrow to it's target. Convergence of sequences are the first topic of inquiry. Bounded, monotone, subsequence, Cauchy convergence are covered. After this limits are introduced and this is not standard in the teaching of calculus but is, in the reviewer's opinion, a more natural way to understand limits, than to introduce them before series and sequences, as is usually the case. With limits (defined rigorously using the standard epsilon and delta definitions) comes continuity and the intermediate value theorem (although no application is given). Differentiation is introduced with a clever story (Zeno, Gottfried and Isaac) of a speeding motorist which seemed to the reviewer a more natural and intuitive method than the standard graphical, secant converging to tangent method. Within the approach given is hidden the fundamental theorem of calculus whereas the graphical method leaves the link between differentiation and integration more mysterious. The product, and chain rules are given for linear functions however, without instruction, it is unlikely the reader will put this together with the local linearity of functions to give the product and chain rule on general functions. The mean value theorem is cleverly applied to prove that the motorist, Zeno, was, in fact, speeding at some point in his journey! Integration is introduced, again with a vehicle travelling. Calculating the distance traveled from the known speed of the vehicle is a more intuitive approach than the common, area under a curve approach. There are many exercises asking the student to write areas, volumes, pressure, work and distance as sums giving the reader insight into the integral being a sum over a continuous domain. The fundamental theorem and techniques of integration are covered briefly. The chapter concludes with a section entitled "From vague to precise", reiterating the process of distilling out definitions, making conjectures and proving them.

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*Classification:* E40 E50 H40 I30 I40 I50 K30

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