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Paoletti, Teo J.
Are all infinities created equal?
Math. Teach. (Reston) 107, No. 2, 98-103 (2013).

From the text: Can one infinity be more than another infinity? Ask students this question, and many will be puzzled; others will insist that "infinity is infinity." The question seems to pique their interest and provides an opportunity to present the beautifully simple but counterintuitive proofs concerning the size of infinity first constructed by Georg Cantor. Such a question may not appear to fit within certain specific content, but the concept of infinity emerges within many contexts throughout the secondary school curriculum, from first-year algebra to BC calculus. Two other topics in the secondary school curriculum closely related to infinity are one-to-one and onto functions, which have clear connections to Cantor's work. A function is "one-to-one" if each output value is the image of a unique input; a function is "onto" if each element in the range is an output for some input value from the domain. Too often, students get little exposure to one-to-one and onto functions. The significance of infinity is supported by NCTM's standards [*National Council of Teachers of Mathematics*, Principles and standards for school mathematics. Reston, VA: NCTM (2000)], which underscore the importance of developing a deep understanding of very large numbers as well as comparing and contrasting the properties of different number systems at the secondary school level. Further, these standards emphasize the importance of students analyzing and evaluating the mathematical thinking, arguments, and proofs of others. A historical exploration of Cantor's proofs concerning infinite sets is one way in which teachers can develop students' understanding of different sets of numbers while allowing students to evaluate and critique the mathematical arguments of others.

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