Pang, Ming Fai; Marton, Ference; Bao, Jian-sheng; Ki, Wing Wah
Teaching to add three-digit numbers in Hong Kong and Shanghai: illustration of differences in the systematic use of variation and invariance.

Summary: In this study, the systematic use of variation and invariance in the teaching of mathematics is examined in accordance with two different but compatible explicit frameworks. We consider the following: (1) differences in tasks that follow on from each other can significantly change what can be learned; (2) Chinese teachers (and probably those in other high achieving countries in South-East Asia) pay considerable attention to the differences between tasks that follow on from each other; and (3) nonetheless, experienced teachers inspired by the two frameworks both of which pay attention to how tasks that follow on from each other can differ, while teaching the same mathematical topic, may still generate very different patterns of tasks and thereby make very different kinds of learning possible. Two experienced Hong Kong teachers devised and planned a series of lessons on the addition and subtraction of three-digit numbers, together with an expert on an explicit framework for systematic use of variation and invariance. Two Shanghai teachers did the same, together with an expert on a different, but compatible, framework. This paper focuses on the analysis of two lessons, one in Hong Kong and one in Shanghai, both being the first of their respective series. The video-recorded lessons were analysed with the aim of illustrating similarities and differences in terms of the systematic use of variation and invariance. These were not intended to be model lessons illustrating the respective frameworks of variation, but rather were examples of the ways teachers consciously and systematically make use of variation and invariance, as inspired by the two frameworks.