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Split-quaternions and pseudo-Pythagorean quintuples.

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From the text: This article deals with a new number system, the split-quaternions. This is an extension of the concept, familiar to students, of taking the real numbers and using them to build a new number system, the complex numbers. In addition, students are familiar with Pythagorean triples, and this paper deals with several higher-dimensional extensions of them. This material may be appropriate for an undergraduate mathematics major or a college mathematics club project. In a previous paper, it was shown that Gaussian integers (complex numbers with integer components) can be used to generate Pythagorean triples. It was further shown that quaternions (a four-dimensional analog of the complex numbers) can be used to generate Pythagorean quintuples (a five-dimensional analog of Pythagorean triples). It seemed like the next natural step would be to consider the split-quaternions, a number system related to, but different from, the quaternions, and their analogous connection to Pythagorean n-tuples, if any. It will be shown that these split-quaternions can be used to generate pseudo-Pythagorean quintuples, which are related to but different from Pythagorean quintuples. The split-quaternions are isomorphic to the real-valued 2×2 matrices under addition and multiplication. The split-quaternions, and that set of matrices, are 4-dimensional algebras with zero-divisors. Split-quaternions also have applications in relativistic physics. After a review of the use of complex numbers to generate Pythagorean triples, we'll look at the split-quaternions and use them to generate pseudo-Pythagorean quintuples and numbers satisfying Diophantine equations of the forms $a^2 + b^2 + c^2 = d^2 + e^2$, $a^2 + b^2 + c^2 = d^2$, and $a^2 + b^2 = c^2 + d^2$.

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