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Introduction to applied geometry. (Einführung in die angewandte Geometrie.)

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This booklet offers an elementary insight into two special domains of geometry, namely analytic and computational geometry. The analytic part provides the underlying structure being the projectively closed (real) finite dimensional Euclidean space and contained figures: lines, hyperplanes, segments of lines and circles; the reader is assumed to be familiar with these concepts. An introductory chapter on homogeneous coordinates and geometric transformations is followed by the first steps into computational geometry: circle splines, skeletons, Delaunay triangulation, Voronoi diagram, algorithm of de Casteljau, convex hull, Bézier splines, control and Farin points. The computational complexity of the mentioned problems is also estimated by the time and space required. In the final chapter the authors show how the different geometries are embedable into projective geometry according to Klein's Erlangen program. This beautiful and well-arranged booklet (for instance theorems are underlaid with pallid violet color) is conceived as university textbook for students of mathematics, informatics, mechanical and constructional engineering at an undergraduate level. The text is divided into 5 chapters, each begins with a short overview and ends with exercises. Chapter 1 (Coordinates and transformations) deals with Cartesian and homogeneous coordinates, hyperplane, hyperplane at infinity, intersection, joining, parallelity, duality, the theorems of Pappus and Desargues, theorem of Szemerédi-Trotter, orientation of three points of a plane, intersection of three linear segments, robustness; geometric transformations: projective and affine mappings, and (Euclidean) similarities. The topics of Chapter 2 (Euclidean geometry) are (direct and indirect) motions and similarities, degree of freedom, orthogonal matrix, classification of the direct motions in 2- and 3-space, helix; circle splines: biarc, interpolation of nonplanar biarcs, inversion with respect to a 1- or 2-sphere, skeleton of simple forms, definition of the $\mathcal{O}$-notation (big O notation), "the skeleton of a polygon with n edges can be computed in $\mathcal{O}(n\log n)$ time and with $\mathcal{O}(n)$ space; as edges of the polygon are admitted linear and circular segments", definition and properties of the Delaunay triangulation, edge flip, empty circle property, "the Delaunay triangulation of a set of n points in the plane is computable in $\mathcal{O}(n\log n)$ time and with $\mathcal{O}(n)$ space"; the Voronoi diagram, duality between Voronoi diagram and Delaunay triangulation of a plane point set; and triangulations in 3-space. Chapter 3 (Affine geometry) treats affine mappings, degree of freedom, barycentric coordinates, the (division) ratio of three distinct collinear points, polynomial curve, control points of a polynomial curve, Lagrange polynomial, Bernstein polynomial, condition for the affine invariance of the description of polynomial curves of degree g via the partition of unity, Bézier curves and some of their useful properties; the blossom concept, the uniqueness of the blossom and the algorithm of de Casteljau, Bézier splines, contact of order k; convex hull and the Graham scan algorithm for point sets in the plane, "the convex hull of n points in the plane is computable in $\mathcal{O}(n\log n)$ time and with $\mathcal{O}(n)$ space"; some facts about the convex hull in 3-space and in higher dimensional spaces; computation of the convex hull with the principle "divide and conquer", and computation of the convex hull by iterative addition. Chapter 4 (Projective geometry) deals with projective mappings, cross ratio, rational curves, rational Bézier curves, quadrics; tangential hyperplane of a quadric, polarity, polarity of an imaginary quadric, classification of quadrics in the real projective 2- and 3-space, the theorems of Pascal, Brianchon, and Steiner; embedding affine geometry into projective geometry: hyperplane at infinity, center of a quadric, classification of quadrics in the real affine 2- and 3-space; embedding Euclidean geometry into projective geometry: circular points, imaginary regular quadric, theorem of Laguerre; and elliptic and hyperbolic geometry. Chapter 5 (Recommended references and appendices) presents a very short presentation of the 14 given references. In Appendix A hints concerning the exercises and in Appendix B a list of notations are given *Rolf Riesinger (Wien)*

Classification: G15
Keywords: Pappus' theorem; Desargues' theorem; theorem of Szemerédi-Trotter; circle spline; biarc; inversion with respect to a 1- or 2-sphere; skeleton; Delaunay triangulation; Voronoi diagram; big O notation; Lagrange polynomial; Bernstein polynomial; partition of unity; Bézier curves; blossom; algorithm of de Casteljau; Bézier splines; 2D Graham scan algorithm; "divide and conquer" principle; algorithm of iterative addition; quadric; Pascal's theorem; Brianchon's theorem; Steiner's theorem; theorem of Laguerre; Cayley-Klein geometry; elliptic geometry; hyperbolic geometry
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