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Mathematical structures. From linear algebra over rings to geometry with sheaves. (Mathematische Strukturen. Von der linearen Algebra über Ringen zur Geometrie mit Garben.)
Heidelberg: Springer Spektrum (ISBN 978-3-662-48869-0/pbk; 978-3-662-48870-6/ebook). x, 303 p. (2016).

The book under review is geared toward upper-level undergraduate students who are familiar with the basics of real analysis and linear algebra. Its main goal is to provide both a panoramic overview and a profound introduction concerning a number of modern, more advanced mathematical concepts permeating contemporay mathematics as a whole. In this regard, the presentation of the interrelation between various mathematical disciplines is particularly emphasized, in the course of which the discussion of several fundamental mathematical structures serves as the guiding methodological principle. As for the precise contents, the book consists of three parts, each of which is divided into several chapters and subsections. Part I is titled "Algebraic structures" and contains four chapters. Chapter 1 is devoted to basic ring theory, whereas chapter 2 discusses modules over a ring,some of their fundamental structural properties, and concrete applications of the latter to linear mappings, including the Jordan normal form of matrices. Chapter 3 develops the principles of multilinear algebra for modules over a ring, with the focus on tensor products and their universal properties, tensor algebras, symmetric algebras, and exterior algebras. Chapter 4 explains how the concrete algebraic structures in chapters 2 and 3 are reflected in the more general conceptual framework of universal algebra and category theory, with a special view toward limits, colimits, and adjoint functors. Part II is superscribed "Local structures" and contains the subsequent three chapters. Chapter 5 provides an introduction to the ubiquitous toolkit of sheaf theory, both from the categorical and from the topological point of view. This includes étale spaces, ringed spaces, and sheaves of modules, thereby offering a glimpse of modern algebraic geometry along the way. Chapter 6 turns to special topological structures, more precisely to differentiable manifolds, tangent and tensor bundles, differential forms, integration theory on real manifolds, and the rudiments of complex analytic functions of one variable. Finally, chapter 7 returns to algebraic geometry by discussing algebraic sets, algebraic varieties, and algebraic schemes in greater detail. Part III offers an outlook to some concepts obtained by combining different types of structures. Its only chapter (Chapter 8) is titled "Additional structures" and touches upon Riemannian manifolds, symplectic manifolds, Kähler manifolds, and Poisson structures. Furthermore, affine connections on manifolds, differential fiber bundles, and group objects in categories serve as supplementing, illustrating examples in this context. All together, this is a very useful book for students seeking orientation for their further specialization in mathematics. The presentation of the material is utmost lucid, sufficiently detailed, versatile, and didactically refined. As such, this excellent primer is a perfect source for further, more detailed reading, and a highly useful companion for students in general. *Werner Kleinert (Berlin)*

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