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Linear algebra as an introduction to abstract mathematics.

Hackensack, NJ: World Scientific (ISBN 978-981-4730-35-8/hbk; 978-981-4723-77-0/pbk). x, 198 p. (2016).

The object of this book is "to introduce abstract mathematics and proofs in the setting of linear algebra to students for whom this may be the first step toward advanced mathematics". Assuming some calculus and only a very basic background in linear algebra (solution of linear equations and manipulation of matrices), it provides a one-semester course in finite-dimensional vector spaces and linear transformations over the real and complex numbers up to the spectral decomposition theorem for normal operators. The main part of the book consists of eleven chapters, each roughly 10–12 pages. The chapter headings are as follows: What is linear algebra? Introduction to complex numbers (operations, polar form and geometric interpretation); Fundamental theorem of algebra (proof of the theorem, factoring polynomials over $\mathbb{C}$); Vector spaces (spaces over $\mathbb{R}$ and $\mathbb{C}$, subspaces and direct sums); Span and bases (linear independence, dimension); Linear maps (null space and range, dimension formula, matrices of linear maps with respect to bases); Eigenvalues and eigenvectors (invariant subspaces, existence of eigenvalues over $\mathbb{C}$, diagonal matrices, every complex linear operator has a triangular matrix over a suitable basis); Permutations and determinants (sign of permutation, definition of determinant, cofactor expansion); Inner product spaces (norms, orthogonality, Gram-Schmidt orthogonalization, orthogonal projections and minimization); Change of bases (change of basis matrix for orthogonal bases); Spectral theorem (Hermitian and unitary operators, normal operators, spectral theorem and diagonalization, positive operators, singular value decomposition). Each of the chapters is followed by "calculational exercises" to test the reader's understanding of the material and by "proof-writing exercises" to develop the reader's ability to construct mathematical arguments of increasing difficulty. Perhaps unusual in a book at this level is a proof of the fundamental theorem of algebra based on the extreme value theorem for real-valued functions of two real variables, and a proof of the existence of eigenvalues for complex linear transformations without the use of determinants, following *S. Axler* [Linear algebra done right. 3rd ed. Cham: Springer (2015; Zbl 1304.15001)]. The use of Axler's approach means that an instructor could omit the chapter on permutations and determinants without affecting the remainder of the book, and indeed, the chapter on determinants is the least well written and may well have been added as an afterthought. The reviewer has always been challenged to know how to deal with the topic of determinants in a course such as this. On one hand, determinants are ubiquitous in the mathematical literature and the formula for a determinant in terms of permutations is theoretically important, if not always essential, in many arguments. On the other hand, this unintuitive formula is clumsy and does not lead to conceptual proofs of fundamental properties of determinants such as the multiplicative property. The last third of the book consists of four appendices of which Appendix A, Supplementary notes on matrices and linear systems, is by far the longest. Appendix A deals at length with the relationship between linear transformations and matrices, Gaussian elimination, elementary matrices, LU-factorization and solution of linear equations. Appendices B and C summarize facts about set theory and algebraic structures. Appendix D explains some history of mathematical notation, use of logical symbols and who first introduced the symbol $i = \sqrt{-1}$, for example. If you plan to teach a course on linear algebra which emphasizes the theoretical side of the subject, you might well consider this book.

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Classification: H65
Keywords: textbook; linear equation; vector space; linear transformation; fundamental theorem of algebra; bases; null space; range; eigenvalue; eigenvector; invariant subspace; determinant; norm; inner product space; orthogonality; Gram-Schmidt orthogonalization; Hermitian operator; unitary operator; normal operators; diagonalization; positive operators; singular value decomposition; Gaussian elimination; LU-factorization
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