

Zbl 1200.35214

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**Optimal Neumann control for the two-dimensional steady-state Navier-Stokes equations.** (English)

Fursikov, Andrei V. (ed.) et al., New directions in mathematical fluid mechanics. The Alexander V. Kazhikhov memorial volume. Boston, MA: Birkhäuser. Advances in Mathematical Fluid Mechanics, 193-221 (2010). ISBN 978-3-0346-0151-1/hbk

The paper focuses on an optimal control problem for a 2D stationary Navier-Stokes system. The authors consider a domain  $\Omega$  of  $\mathbb{R}^2$  consisting of a rectangle without a subset bounded by a smooth curve  $S$ . The Navier-Stokes system  $-\Delta v + v \cdot \nabla v + \nabla p = 0$ ,  $\nabla \cdot v = 0$  is considered in  $\Omega$ . Dirichlet or Neumann boundary conditions are imposed on the different pieces of the boundary of this domain. The optimal control problem consists to minimize the functional  $J = \int_S n \cdot \sigma \cdot e_1 dx$  under the action of controls  $u^1$  and  $u^2$  imposed on subintervals of the horizontal parts of the boundary of the domain  $\Omega$  through Neumann boundary conditions. Here  $e_1$  is the first unit vector of  $\mathbb{R}^2$  and  $n \cdot \sigma = -pn + 2D(v)n$ , where  $n$  is the unit outer normal and  $2D(v) = (\partial_j v_i + \partial_i v_j)_{i,j=1,2}$ . The applied controls are supposed to satisfy  $\|u^1\|^2 + \|u^2\|^2 \leq \gamma^2$  in the  $L^2$ -norm of their respective domains, for some positive constant  $\gamma$ . The first main result of the paper proves the existence of a generalized solution of this optimal control problem. The authors start proving further properties of the generalized solution of the associated Stokes problem. They also prove the existence of a generalized solution of the Navier-Stokes problem assuming the existence of admissible collections  $(v, p, u^1, u^2)$  for this Navier-Stokes problem and that the boundary data are small enough. This is done building a contraction operator associated to this Navier-Stokes problem. The proof of the existence of an optimal solution is obtained rewriting the original problem as a minimization problem for a continuous functional on a compact set. The second main result of the paper establishes the optimality system for the optimal solution. The authors here use the abstract Lagrange principle framework. The paper ends with the presentation of some briefly described numerical simulations.

*Alain Brillard (Riedisheim)**Keywords* : optimal control problem; stationary 2D Navier-Stokes system; generalized solution; minimization of drag; abstract Lagrange principle; Lagrange multiplier; optimality system*Classification* :

- \*35Q30 Stokes and Navier-Stokes equations
- 76D55 Flow control and optimization
- 76D05 Navier-Stokes equations (fluid dynamics)
- 49J20 Optimal control problems with PDE (existence)
- 49K20 Optimal control problems with PDE (nec./ suff.)
- 93C20 Control systems governed by PDE