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Zbl 1195.46059

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Finite-dimensional Hilbert C^*-modules. (English)

Banach J. Math. Anal. 4, No. 2, 147-157, electronic only (2010). ISSN 1735-8787/e
emis:journals/BJMA/tex_v4_n2_a13.pdf
eudml:232841
http://projecteuclid.org/bjma
http://www.emis.de/journals/BJMA/

In studying perturbations of the Wigner equation in inner product C^*-modules, *J. Chmieliński, D. Ilišević, M. S. Moslehian* and *Gh. Sadeghi* [J. Math. Phys. 49, No. 3, 033519, 8 p. (2008; Zbl 1153.81342)] introduced the condition [H] stating that, for every bounded sequence (v_n) in a Hilbert C^*-module V, there are a subsequence (v_{n_k}) of (v_n) and $v \in V$ such that, for every $y \in V$, $\lim_{k\to\infty} \|\langle y, v_{n_k}\rangle - \langle y, v\rangle\| = 0$. They proved that condition [H] is satisfied in every Hilbert C^*-module over a finite-dimensional C^*-algebra. Later, *Lj. Arambašić, D. Bakić* and *M. S. Moslehian* [Oper. Matrices 3, No. 2, Article ID 14, 235–240 (2009; Zbl 1188.46036)] proved that, if a full Hilbert A-module satisfies condition [H], then A must be finite-dimensional.
In the paper under review, the authors characterize the finite-dimensional Hilbert C^*-modules in terms of the convergence of certain sequences. More precisely, they prove that, if V is a full right Hilbert module over a C^*-algebra A, then the following statements are mutually equivalent: (i) V is finite-dimensional; (ii) A and the C^*-algebra $K(V)$ of compact operators on V are finite-dimensional; (iii) for every bounded sequence (v_n) in V, there are a subsequence (v_{n_k}) of (v_n) and $v \in V$ such that $\lim_{k\to\infty} \|v_{n_k}a - va\| = 0$ $(a \in A)$ and $\lim_{k\to\infty} \|\langle y, v_{n_k}\rangle - \langle y, v\rangle\| = 0$ $(y \in V)$; (iv) $K(V)$ is a unital C^*-algebra, and for every bounded sequence (v_n) in V, there are a subsequence (v_{n_k}) of (v_n) and $v \in V$ such that $\lim_{k\to\infty} \|\langle y, v_{n_k}\rangle - \langle y, v\rangle\| = 0$ $(y \in V)$; (v) A is a unital C^*-algebra, and for every bounded sequence (v_n) in V, there are a subsequence (v_{n_k}) of (v_n) and $v \in V$ such that $\lim_{k\to\infty} \|v_{n_k}a - va\| = 0$ $(a \in A)$.

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Keywords: C^*-algebra; Hilbert C^*-module; (weakly) compact operator; finite-dimensional C^*-algebra; finite-dimensional Hilbert C^*-module
Classification :
∗46L08 C*-modules
46L05 General theory of C*-algebras
46C50 Generalizations of inner products