

**Zbl 1201.47038****Heath, Matthew J.****Compact failure of multiplicativity for linear maps between Banach algebras.**

(English)

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[emis:journals/BJMA/tex\\_v3\\_n2\\_a14.pdf](https://emis.journals/BJMA/tex_v3_n2_a14.pdf)[eudml:228750](https://eudml.org/228750)<http://projecteuclid.org/bjma><http://www.emis.de/journals/BJMA/>Let  $\mathcal{A}$  and  $\mathcal{B}$  be Banach algebras and let  $T : \mathcal{A} \rightarrow \mathcal{B}$  be a linear map. Set

$$S_T(a, b) = T(ab) - T(a)T(b), \quad a, b \in \mathcal{A}.$$

For  $\delta > 0$ , the map  $T$  is called  $\delta$ -multiplicative if  $\|S_T\| < \delta$ . In *B. E. Johnson* [J. Lond. Math. Soc., II. Ser. 34, 489–510 (1986; Zbl 0625.46059)] and [J. Lond. Math. Soc., II. Ser. 37, No. 2, 294–316 (1988; Zbl 0652.46031)], pairs  $(\mathcal{A}, \mathcal{B})$  which are AMNM (almost multiplicative bounded linear maps are near multiplicative bounded linear maps) were investigated. Since then, many authors have contributed to the study of these algebras. In the paper under review, the author considers other concepts of smallness of  $S_T$ . First of all, he introduces notions of compactness and weak compactness for multilinear maps from a product of normed spaces to a topological space. As pointed out by the author, compact multilinear maps were also considered in the normed case by *N. Krikorian* [Proc. Am. Math. Soc. 33, 373–376 (1972; Zbl 0235.46068)]. Now a map  $T$  is said to be a cf-homomorphism (compact from homomorphism) if  $S_T$  is compact.

On the other hand,  $T$  is called semi-cf-homomorphism if, for each  $a \in \mathcal{A}$ ,  $S_T(a, \cdot)$  and  $S_T(\cdot, a)$  are compact linear maps. In a similar way, the author defines weakly compact,  $n$ -dimensional (resp., semi weakly compact, semi  $n$ -dimensional) from homomorphism. The author studies general properties of such maps. Moreover, he gives a characterization of some Banach function algebras where such maps are automatically multiplicative. Finally, the paper is concluded with generalizations of some results in the Hochschild-Kamowitz cohomology theory.

*Nadia Boudi (Meknes)**Keywords* : Banach algebra; compact multilinear map; Banach extension*Classification* :

- \*47B48 Operators on Banach algebras
- 46H05 General theory of topological algebras
- 46J10 Banach algebras of continuous functions
- 46H25 Topological modules