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The arithmetic of Prym varieties in genus 3. (English)

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Let $\pi : D \to C$ be an unramified finite morphism of degree two between curves over a field K. The Prym variety of D/C is the connected component of the identity element of the kernel of the map $\pi_* : \mathrm{Jac}(D) \to \mathrm{Jac}(C)$, denoted by $\mathrm{Prym}(D/C)$. In his earlier work, the author considered this setup where C is a hyperelliptic curve, and had success in applying the arithmetic theory of this setup, combined with explicit Chaubauty methods, to determining the set of rational points on such curves. In the paper under review, he considers the case where C is non-hyperelliptic of genus 3, and accomplishes the following:
(1) An explicit construction of a curve F of genus 2 for which $\mathrm{Jac}(F) \cong \mathrm{Prym}(D/C)$;
(2) a construction of a map $\phi : D \to \mathrm{Jac}(F) \cong \mathrm{Prym}(D/C)$ which does not require a rational point on D.
When $K = \mathbb{C}$, the description $\mathrm{Prym}(D/C) \cong \mathrm{Jac}(F)$ can be found in [*E. Arbarello, M. Cornalba, P. A. Griffiths*, and *J. Harris*, Geometry of algebraic curves, Vol. I, Grundlehren der mathematischen Wissenschaften, 267. New York etc.: Springer-Verlag (1985; Zbl 0559.14017)]. When D has a rational point, we have the well-known Abel-Prym map $(\mathrm{id}_* - \iota_*) : D \to \mathrm{Prym}(D/C)$ where ι is the nontrivial involution on D with no fixed points. Combined with earlier work, the construction of a curve F described above gives a complete description of how principally polarized Abelian surfaces arise as Prym varieties with $g(C) = 3$ over base fields of characteristic zero which is not necessarily algebraically closed.
The author also proves a case of the converse: Given an arbitrary curve F of genus 2 over a number field K, there is a Prym variety $\mathrm{Prym}(D/C)$ which is isomorphic to $\mathrm{Jac}(F)$, and he remarks that the construction of $\mathrm{Prym}(D/C)$ certainly works over any base field of odd characteristic with sufficiently many elements.
As an application the author presents three examples of non-hyperelliptic curves $C/\mathbb{Q}$ of genus 3: (1) A curve C with exactly one rational point; (2) An unramified double cover $D/\mathbb{Q}$ of C such that both curves have points locally everywhere but have no rational points; (3) A curve C with all 28 bitangents rational, for which he uses his general form of a non-hyperelliptic curve of genus 3 and a Kummer surface $\mathrm{Jac}(F)/\langle \pm 1\rangle$.

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Keywords : Prym varieties; Chabauty methods; rational points on curves; covering technique; Brauer-Manin; smooth plane quartics
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