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Calegari, Danny

Foliations and the geometry of 3-manifolds. (English)

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This is a book on various subjects related to low-dimensional topology, especially, to 3-maifolds. The main goal (according to the author's words) is to present a "pseudo-Anosov theory of 3-manifolds." The material covered includes classical as well as recent results, and the objects of study are embedded surfaces, geometric structures, foliations, laminations, homeomorphisms, mapping classes, and other notions related to 3-manifolds. Several notions introduced by Bill Thurston in the last 30 years or so are present in all the theories that are contained in this book, and Thurston's mark is apparent in almost every page.

The book is extremely well written and it is very pleasant to read. The definitions and the statements of the results are presented clearly, with a lot of illustrations and judicious examples. Of course, part of the results are stated without proofs, but there are also important results that are given with detailed proofs, sometimes due to the the author himself, simplifying or filling some gaps in existing proofs. Some of the results (especially those of Chapter 8) are due to the author.

This book is unique on most of the topics that it contains, and, for this and for other reasons, it constitutes a very important contribution to low-dimensional topology literature. The book will be very useful to anyone working in the subject.

The following is a quick review of the topics considered in this book.

Chapter 1 is an overview on surfaces, their geometric structures and their mapping class groups. It includes an introduction to 2-dimensional hyperbolic geometry, measured foliations, laminations, train tracks, pseudo-Anosov theory and mapping tori.

Chapter 2, called "The topology of S^1", is an exposition of several subjects related to homeomorphisms of the circle and of the interval, and, more generally, to one-dimensional dynamics. The objects studied in this chapter are used in later parts of the book. These objects include laminations on the disk, monotone maps and monotone equivalence of group actions, Denjoy theory and the work of Ghys on rotation numbers, Mather and Thurston's result on the cohomology of the group of orientation-preserving homeomorphisms of the circle, Thurston's stability theorem for the group of orientation-preserving homeomorphisms of the interval, amenability, bounded cohomology and the Milnor-Wood inequality, a review of the works of Gabai and of Casson-Jungries on convergence groups, Thompson's groups, Godbillon-Vey cocycles (work of Duminy and of Sergiescu) and several other related objects.

Chapter 3 is a review of the theory of minimal surfaces, especially in 3-manifolds. The exposition starts with a short introduction to the fundamental objects in this theory (connections, curvature, mean curvature, the second fundamental form, stable and least area surfaces, and so on). Then, the author surveys the general existence theorems of Douglas and Rado and of Schoen-Yau, together with the important results on least-area minimal embeddings of spheres by Meeks, Simon and Yau, the compactness theorems of Choi and Schoen, and the strengthened solution to the classical Plateau problem by Meeks and Yau.

Chapter 4 is about taut codimension-one foliations in 3-manifolds and, according to the author, this is the central subject of the book. A foliation is said to be taut if every leaf is intersected by a circle which is transverse to the foliation. The study of taut foliations is a classical subject. Existence theorems for taut foliations imply classical results such as Alexander's theorem stating that every tame sphere in $\mathbb{R}^3$ bounds a ball. The chapter starts with a general introduction to foliation theory, in particular, the theory of foliated bundles, holonomy and Reeb stability. In particular, there is a presentation of a construction by Thurston of foliations on triangulated 3-manifolds, with a mention of Thurston's "jiggling lemma". The author then presents classical results of Novikov, Rosenberg, Palmeira, and he discusses Anosov flows and foliations of circle bundles, in particular Seifert fibered spaces.

In Chapter 5, the author studies a special class of taut foliations, which are foliations of finite depth. The depth of a leaf of a foliation is defined inductively as follows. Let λ be a leaf in a foliation. Then, λ is said to be of depth zero it is closed. If $\overline{\lambda} - \lambda$ consists of leaves of depth at most n, then λ is of depth at most $n + 1$. The foliation is of depth n if every leaf is of depth at most n, with n minimal. The theory of finite depth foliations parallels the theory of Haken manifolds. After a presentation of the theory of Thurston's norm, the author presents the work of Roussarie and Thurston on the homotopy of surfaces in Reebless foliations, and an important inequality due to Thurston relating the Thurston norm of a surface immersed in a manifold equipped with a taut foliation with the value on that surface of the Euler class of the tangent bundle of the foliation. The author then presents an existence theorem due to Gabai of transverse taut foliations on sutured manifolds. The chapter also contains a discussion of algorithmic problems related to minimizing Thurston's norm.

In Chapter 6, the author discusses branched surfaces, essential laminations and genuine laminations. He presents results of Mosher and an unpublished result due to Gabai and to Mosher on the existence of pseudo-Anosov flows whose stable and unstable laminations are transverse to a given finite-depth foliation on a closed, oriented, irreducible, atoroidal 3-manifold.

Chapter 7 concerns again taut foliations. The author presents Candel's uniformization theorem, which gives necessary and sufficient conditions for a lamination whose leaves are compact Riemann surfaces to admit a leafwise hyperbolic structure. This theorem implies that given a taut foliation F on an atoroidal manifold M, we can find a metric on M such that every leaf of F, equipped with the induced length metric, is hyperbolic. This implies that every leaf λ of the induced universal cover foliation $\widetilde{F}$ is isometric to the hyperbolic plane $\mathbb{H}^2$; therefore the leaf λ has an associated circle at infinity. The collection of circles at infinity associated to the various leaves λ can be identified to a single *universal circle* which gives information on the tangential geometry and on the transverse topology of $\widetilde{F}$. The idea of a universal circle was introduced by Thurston, and it was developed in a slightly different form (presented in this book) by Calegari-Dunfield.

In Chapter 8, the author shows that if F is a taut foliation on an atoroidal 3-manifold M, then the construction of the universal circle gives rise to a pair of essential laminations transverse to F.

Chapter 9 concerns slitherings, for which the main reference is again a paper by Thurston. A 3-manifold M *slithers over* S^1 if the universal cover $\widetilde{M}$ fibers over S^1 in such a way that $\pi_1(M)$ acts on $\widetilde{M}$ by bundle automorphisms. The foliation of $\widetilde{M}$ by the connected components of the fibers descends on M as a foliation.
Chapter 10, entitled "Peano curves", makes fascinating relations between various objects considered in the previous chapters, associated to a hyperbolic 3-manifold equipped with a taut foliation, namely, universal circles, pairs of laminations transverse to the foliation, and the sphere at infinity of the universal cover. The author's approach makes use of the universal Teichmüller space, there are relations with Julia sets of rational maps of the sphere, and with Moore's approximation theorem. The author applies these ideas to quasi-geodesic flows, in particular, to quasi-geodesic Anosov flows. The book ends with mentioning very recent work of Fenley on ideal boundaries of pseudo-Anosov flows.

Athanase Papadopoulos (Strasbourg)

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Classification :
∗57-02 Research monographs (manifolds)
57M50 Geometric structures on low-dimensional manifolds
37E10 Maps of the circle
37E45 Rotation numbers and vectors
57R30 Foliations; geometric theory
37D20 Uniformly hyperbolic systems
57N10 Topology of general 3-manifolds