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**Zbl 0815.35002****Colombeau, Jean François****Multiplication of distributions. A tool in mathematics, numerical engineering and theoretical physics.** (English)

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The objective of this book is to present a recent mathematical tool heuristically with emphasis on algebraic calculations and numerical recipes. These have been used to obtain numerical solutions of the systems of equations modelling elasticity, elastoplasticity, acoustic diffusion, multifluid flows, etc. which contain products looking like “ambiguous multiplication of distributions” (of the kind  $Y\delta$  where  $Y$  is the Heaviside function). These cannot be tackled by theory of distribution of Schwartz due to the “impossibility of the multiplication of distributions” proved by Schwartz in 1954. The idea enunciated is that these statements of equations of physics are basically sound and that a new mathematical theory of generalized functions is needed to explain and tackle these. For the development of this theory one can refer to *J. F. Colombeau* [New generalized functions and multiplication of distributions (1984; Zbl 0532.46019)] and *H. A. Biagioni* [A nonlinear theory of generalized functions (1990; Zbl 0694.46032)]. The theoretical consequences of this mathematical tool are not discussed in this monograph.

This new theory resolves the ambiguities appearing in equations of physics involving “heuristic multiplication of distributions” by giving more precise formulations of the equations which does not make sense in (Schwartz’s) distribution theory. This technique gives new algebraic formulas and new numerical schemes.

Part I, consisting of the first two chapters, deals with preliminaries from mathematics and physics. Part II, consisting of Chapter 3, briefly introduces the new theory of generalized functions and gives solutions for previously unsolvable equations. This theory is consistent with classical analysis. Since the limitations of this new theory are unknown yet, various research directions have been proposed.

Let  $\Omega$  be any open set in  $\mathbb{R}^n$ . A generalized function  $G$  on  $\Omega$  is an “ideal limit”, when  $\varepsilon \rightarrow 0$ , of a family  $\{R_\varepsilon\}_{0 < \varepsilon < 1}$  (which is called a representation of  $G$ ) of  $C^\infty$  functions on  $\Omega$ . There is a natural inclusion of  $C^\infty(\Omega)$  into  $\mathcal{G}(\Omega)$  (set of new generalized functions) with  $C^\infty(\Omega) \subset D'(\Omega) \subset \mathcal{G}(\Omega)$  and  $\mathcal{G}(\Omega)$  is a differential algebra. All the classical operations applicable in  $C^\infty(\Omega)$ , the addition, scalar multiplication and the multiplication of functions, extend to  $\mathcal{G}(\Omega)$ .

Shock waves are “very quick” variations of the physical variables and are represented by generalized functions which contain some piece of information on the jumps. A basic role is played by a relation on  $\mathcal{G}(\Omega)$ , called association (weak equality). This is an equivalence relation. However, this relation is not compatible with multiplication.

The importance of this new theory is illustrated, for example, in discussing the discontinuous solutions of the equation  $U_t + UU_x = 0$ . It is shown that the strong formulation (Strong equality in  $\mathcal{G}(\mathbb{R}^2)$ ) has no solution whereas the weak form (association in  $\mathcal{G}(\mathbb{R}^2)$ )

is shown to have a solution if the well known jump condition is satisfied. The case of systems in nonconservative form is also discussed. The result shows a major difference between the nonconservative case (an infinity of possible jump relations) and the conservative case (a unique jump relation given by the classical Rankine Hugoniat jump formulas).

Part III deals with the new numerical methods for systems containing multiplication of distributions in elastoplasticity, multifluid flows, linear systems of acoustics, etc. For systems in nonconservative form, a way to ensure uniqueness of the jump condition is discussed in chapter 4.1. A general method for the removal of the ambiguity in a system of equations is discussed in chapter 4.3. The method consists in stating the basic laws of physics with (strong) equality in  $\mathcal{G}$  and the constitutive equation with association (weak formulation). Numerical schemes are discussed at length.

Part IV deals with basic heuristic calculations of quantum field theory (the interacting field equations) and a brief mathematical introduction to generalized functions as given in *J. F. Colombeau* [Elementary introduction to new generalized functions (1985; Zbl 0584.46024)].

Research problems, useful remarks and numerical solutions with graphical illustrations constitute the special features of this monograph meant for Physicists and Engineers. A judicious balance between theory and applications is maintained, keeping the objective in mind. It is a valuable and welcome addition to existing literature in this field.

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*Keywords* : association relation; multiplication of distributions; generalized functions

*Classification* :

- \*65M99 Numerical methods for IVP of PDE
- 65N99 Numerical methods for BVP of PDE
- 35D05 Existence of generalized solutions of PDE
- 46F10 Operations with distributions (generalized functions)