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ZMATH 06130636

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The small-maturity smile for exponential Lévy models.

SIAM J. Financ. Math. 3, No. 1, 33-65, electronic only (2012).

Summary: We derive a small-time expansion for out-of-the-money call options under an exponential Lévy model, using the small-time expansion for the distribution function given in [*J. Figueroa-López* and *C. Houdré*, Stochastic Process. Appl. 119, No. 11, 3862–3889 (2009; Zbl 1179.60026)], combined with a change of numéraire via the Esscher transform. In particular, we find that the effect of a nonzero volatility σ of the Gaussian component of the driving Lévy process is to increase the call price by $\frac{1}{2}\sigma^2 t^2 e^k \nu(k)(1+o(1))$ as $t \to 0$, where ν is the Lévy density. Using the small-time expansion for call options, we then derive a small-time expansion for the implied volatility $\hat{\sigma}_t^2(k)$ at log-moneyness k, which sharpens the first order estimate $\hat{\sigma}_t^2(k) \sim \frac{\frac{1}{2}k^2}{t\log(1/t)}$ given in [*P. Tankov*, "Pricing and hedging in exponential Lévy models: review of recent results", Lecture Notes in Mathematics 2003, 319–359 (2011; Zbl 1205.91161)]. Our numerical results show that the second order approximation can significantly outperform the first order approximation. Our results are also extended to a class of time-changed Lévy models. We also consider a small-time, small log-moneyness regime for the CGMY model and apply this approach to the small-time pricing of at-the-money call options; we show that for $Y \in (1,2)$, $\lim_{t\to 0} t^{-1/Y}\mathbb{E}(S_t - S_0)_+ = S_0\mathbb{E}^*(Z_+)$ and the corresponding at-the-money implied volatility $\hat{\sigma}_t(0)$ satisfies $\lim_{t\to 0}\hat{\sigma}_t(0)/t^{1/Y-1/2} = \sqrt{2\pi}\,\mathbb{E}^*(Z_+)$, where Z is a symmetric Y-stable random variable under $\mathbb{P}^*$ and Y is the usual parameter for the CGMY model appearing in the Lévy density $\nu(x) = Cx^{-1-Y}e^{-Mx}\mathbf{1}_{\{x>0\}} + C|x|^{-1-Y}e^{-G|x|}\mathbf{1}_{\{x<0\}}$ of the process.

Classification: 60G51 60F99 91G20 91G60
Keywords: exponential Lévy models; time-changed Lévy models; option pricing; short-time asymptotics; implied volatility
doi:10.1137/110820658