Selected reviews from

Zentralblatt MATH

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Gert-Martin Greuel
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(Editors)

Pelczar, Andrzej

Spellings: Pelczar, Andrzej [73] Pelczar, A. [3]
Author-Id: pelczar.andrzej
Publications: 76 including 71 Journal Article(s)

MSC 2010

22	**54** · General topology
21	**34** · Ordinary differential equations (ODE)
21	**37** · Dynamical systems and ergodic theory
17	**35** · Partial differential equations (PDE)
6	**01** · History; biography

more ...

Journals

19	Bulletin de l'Académie Polonaise des Sciences, Série des Sciences Mathématiques, Astronomiques et Physiques
12	Annales Polonici Mathematici
7	Zeszyty Naukowe Uniwersytetu Jagiellońskiego. Universitatis Iagellonicae Acta Mathematica
2	Annales Societatis Mathematicae Polonae. Seria II. Wiadomości Matematyczne
1	Antiquitates Mathematicae

more ...

Co-Authors

2	Denkowski, Zdzislaw
1	Górecki, Henryk
1	Kisyński, Jan
1	Malczak, Jan
1	Trzepizur, Andrzej

Publication Years

Zbl 102.02603
Pelczar, Andrzej
On the invariant points of a transformation.
Ann. Pol. Math. 11, 199-202 (1961).

Der folgende Fixpunktsatz wird bewiesen: Es sei $P \neq \emptyset$ eine halbgeordnete Menge, in der zu jeder Teilmenge $Q \subset P$, $Q \neq \emptyset$ das Supremum existiert. Es sei weiter V eine Abbildung von P in sich, so daß gilt:

(a) $x, y \in P, x \leqq y \Rightarrow V(x) \leqq V(y)$, und

(b) es gibt ein $z_0 \in P$ mit $z_0 \leqq V(z_0)$.

Dann ist die Menge $A = \{z \mid z \in P, V(z) = z\}$ nicht leer und A enthält ein maximales Element.
[Da die Menge $B = \{z \mid z \in P, z \geqq z_0\}$ ein vollständiger Verband ist und $V(B) \subset B$, ist die Behauptung $A \neq \emptyset$ naheliegend; vgl. *G. Birkhoff*, Lattice theory (1948; Zbl 33,101); S. 54 Theorem 8.] *J. Jakubík*

Zbl 589.34041
Pelczar, Andrzej
Semi-stability of motions and regular dependence of limit sets on points in general semi-systems.
Ann. Pol. Math. 42, 263-282 (1983).

The author, well-known specialist in the theory of abstract dynamical systems, studies dynamical systems and semisystems and some generalizations of them. The results are obtained for semisystems on metric spaces, but some of them can easily be generalized and extended for Hausdorff topological spaces satisfying the first axiom of countability. In the following, we mention only the main results.

In Section 2, limit sets and generalized semistability of motions in semisystems are studied. From Section 4 we retain the following results: Theorem 1. Let $\alpha \geq 0$ and $x \in X$ (X is a metric space). Suppose that the motion Π^x of the semisystem is α-semistable. Assume that $\{x_n\}$ and $\{y_n\}$ are sequences of elements of X such that $x_n \to x$, $y_n \to y$ as $n \to \infty$, $y_n \in \Lambda(x_n) = \{y \in X :$ there is a sequence $\{t_m\}$ of elements of $\mathbb{R}_*$ such that $t_m \to \infty$ and $y = \lim \Pi(t_m, x)\}$ for every n. Then there exists a sequence $\{s_n\}$ of elements of $\mathbb{R}_*$ such that $s_n \to \infty$ as $n \to \infty$ and $\limsup_{n\to\infty} \rho(\pi(s_n, x), y) \leq \alpha$. Theorem 2. If Π^x is semistable then the mapping (*) $X \ni y \mapsto \Lambda(y) \in CL(X)$ is upper semicontinuous at the point x. Theorem 3. Suppose that X satisfies at every point the condition Comp(α) (i.e. for given $\alpha \geq 0$, for each $x \in X$, there is a positive number β_x such that the ball $B(x, \alpha + \beta_x)$ is relatively compact). If Π^x is α-semistable then the mapping (*) is α-upper semicontinuous at the point x. Similar results for the weak α-semistability of motions and nonemptiness of some limit sets are established in Section 5. In Section 6, the regularity of the mapping $x \to \Lambda(x)$ in dynamical semisystems is studied. A typical result in this section is the following: Theorem 7. Assume that X is a locally compact space and $(X, \mathbb{R}_*; \Pi)$ is a dynamical semisystem. Let $x \in X$ be such that for every $\epsilon > 0$ there exist $\delta > 0$ and $u \in \mathbb{R}_*$ such that if $z \in B(x, \delta)$ then $d(\Pi(t, z), \Pi(x)) < \epsilon$, for every $t \geq u$, and for every $\epsilon > 0$ there is $\delta > 0$ such that for every $z \in B(x, \delta)$ and every $t \in \mathbb{R}_*$ there is $u \in \mathbb{R}_*$ for which $\rho(\pi(t + u, x), \pi(t + u, z)) < \epsilon$. If $\{x_n\}$ is a sequence of elements of X such that $x_n \to x$, $\Lambda(x_n) = y_n$ for every n where $y_n \to y$, then $y \in \Lambda(x)$. The proofs of the results are based on the abstract theory of dynamical systems on metric spaces. *N. Luca*

De Lellis, Camillo

Spellings: De Lellis, Camillo [39] de Lellis, Camillo [11] de Lellis, C. [1] De-Lellis, Camillo [1]
Author-Id: de-lellis.camillo
Publications: 51 including 2 Book(s) and 40 Journal Article(s)

MSC 2010

28	**35** · Partial differential equations (PDE)
19	**49** · Calculus of variations and optimal control; optimization
8	**53** · Differential geometry
6	**76** · Fluid mechanics
4	**34** · Ordinary differential equations (ODE)

more ...

Journals

4	Archive for Rational Mechanics and Analysis
3	Communications on Pure and Applied Mathematics
3	Comptes Rendus. Mathématique. Académie des Sciences, Paris
2	Calculus of Variations and Partial Differential Equations
2	Communications in Partial Differential Equations

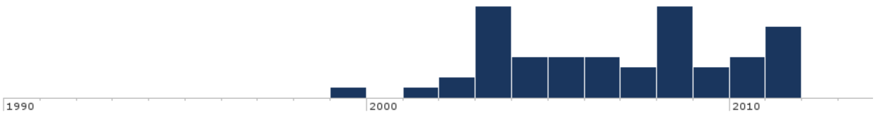
more ...

Co-Authors

5	Ambrosio, Luigi
4	Székelyhidi, László jun.
3	Colding, Tobias Holck
3	Crippa, Gianluca
3	Muller, Stefan C.

more ...

Publication Years



Zbl 1192.35138
de Lellis, Camillo; Székelyhidi, László jun.
On admissibility criteria for weak solutions of the Euler equations.
Arch. Ration. Mech. Anal. 195, No. 1, 225-260 (2010).

The authors consider the Cauchy problem for the incompressible Euler equations in n space dimensions, $n \geq 2$,

$$\begin{aligned} \frac{\partial v}{\partial t} + \operatorname{div}(v \otimes v) + \nabla p = 0, \quad \operatorname{div} v = 0 \quad x \in \mathbb{R}^n,\ t > 0, \\ v(x,0) = v_0(x), \quad x \in \mathbb{R}^n, \end{aligned} \tag{$*$}$$

where v_0 is a given divergence-free vector. The main result of the paper is the non-uniqueness theorem to the problem ($*$). It is proved that there exist bounded and compactly supported v_0 for which there are

(1) infinitely many weak solutions of ($*$) satisfying both the strong and local energy equalities;

(2) weak solutions of ($*$) satisfying the strong energy inequality but not the energy equality;

(3) weak solutions of (∗) satisfying the weak energy inequality but not the strong energy inequality.

Another non-uniqueness result is obtained to the system of isentropic gas dynamics in Eulerian coordinates

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho v) &= 0, \quad x \in \mathbb{R}^n,\ t > 0, \\ \frac{\partial}{\partial t}(\rho v) + \operatorname{div}(\rho v \otimes v) + \nabla[p(\rho)] &= 0, \quad x \in \mathbb{R}^n,\ t > 0, \\ v(x,0) = v_0(x), \quad \rho(x,0) &= \rho_0(x), \quad x \in \mathbb{R}^n. \end{aligned}$$

Here v is the velocity of a gas, ρ is the density, the pressure p is a function of ρ.

The proves are based on the Baire category method.

Il'ya Sh. Mogilevskij (Tver')

Edelsbrunner, Herbert

Spellings: Edelsbrunner, Herbert [131] Edelsbrunner, H. [34]
Author-Id: edelsbrunner.herbert
Publications: 165 including 5 Book(s) and 118 Journal Article(s)

MSC 2010

129	**68** · Computer science
43	**52** · Convex and discrete geometry
18	**65** · Numerical analysis
17	**51** · Geometry
15	**05** · Combinatorics

more ...

Journals

33	Discrete & Computational Geometry
9	SIAM Journal on Computing
7	Information Processing Letters
6	Algorithmica
6	Computational Geometry

more ...

Co-Authors

25	Guibas, Leonidas J.
25	Sharir, Micha
19	Chazelle, Bernard
13	Seidel, Raimund
12	Harer, John L.

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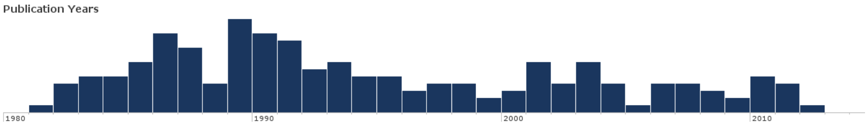
Publication Years



Zbl 1232.55012
Edelsbrunner, Herbert; Morozov, Dmitriy; Patel, Amit
Quantifying transversality by measuring the robustness of intersections.
Found. Comput. Math. 11, No. 3, 345-361 (2011).

Let $\mathbb{X}, \mathbb{Y}$ be topological spaces, $\mathbb{A} \subseteq \mathbb{Y}$ a subspace, and assume that the space of continuous functions from $\mathbb{X}$ to $\mathbb{Y}$ possesses a metric $\|\cdot\|$. An r-perturbation of $f : \mathbb{X} \to \mathbb{Y}$ is a function $h : \mathbb{X} \to \mathbb{Y}$ such that $\|f - h\| \leq r$. The authors of the article define the well module U of f, which encodes topological information about the intersection of the image of f with $\mathbb{A}$, and show that it is stable under perturbations. The well module is constructed

by first defining the map $f_{\mathbb{A}} : \mathbb{X} \to \mathbb{R}$, where $f_{\mathbb{A}}(x)$ is the infimum of all values r such that there exists an r-perturbation h with $h(x) \in \mathbb{A}$, and setting $F(r)$ to be the homology group of the preimage $f_{\mathbb{A}}^{-1}[0, r]$ in $\mathbb{X}$. Next, for each real number r, the subgroup $U(r)$ of $F(r)$ is defined as the intersection over all r-perturbations h of f of the images of the maps in homology induced by inclusion $h^{-1}(\mathbb{A}) \subseteq f_{\mathbb{A}}^{-1}[0, r]$. Finally, the well module U is taken to be the collection of groups $U(r_j)$, where $r_0 < r_1 < \cdots < r_l$ are the critical values of $f_{\mathbb{A}}$; i.e., those r for which the inclusion-induced map $F(r - \delta) \to F(r + \delta)$ is not an isomorphism for any $\delta > 0$. Although the groups $U(r_j)$ do not form a filtration of U, the authors construct one using a generalized form of persistent homology called zigzag persistence. The resulting persistence diagram for U is shown to be stable under perturbations: for any map $g : \mathbb{X} \to \mathbb{Y}$ with corresponding well module V, the bottleneck distance between the persistence diagrams of U and V is bounded by $\|f - g\|$.
The authors assume a general mathematical audience versed in homology; while a previous knowledge of persistent homology is not assumed, it is recommended. The article is well-written and organized, and includes a section of topological applications. *Jason Hanson (Redmond)*

Zbl 1193.55001
Edelsbrunner, Herbert; Harer, John L.
Computational topology. An introduction.
Providence, RI: American Mathematical Society (AMS)
(ISBN 978-0-8218-4925-5/hbk). xii, 241 p. (2010).

There are innumerable books available that discuss topology from a purely theoretical point of view. However, there are currently very few that address the practical computational aspects: the data structures used to represent topological spaces, and the algorithms to compute topological information. *Computational Topology: An Introduction* helps to bridge this gap, and provides a nice overview of computational topology and its applications. The book is divided into three parts. The first part discusses basic topological and geometric concepts, and gives data structures and algorithms related to these concepts. This portion of the book is somewhat encyclopedic, covering a fair number of topics in a short stretch. The discussion starts with one dimensional figures: graphs and planar curves; proceeds on to surfaces: triangulations, immersions, and mesh simplification; and ends with various types of simplicial complexes: Čech, Vietoris-Rips, Delaunay, and alpha. Proofs of relevant results are occasionally given, but more often they are simply stated. The second part of the text gives a treatment of the homology of a simplicial complex. Since only coefficients in the field of two elements are considered, the discussion is somewhat simplified in comparison to a standard treatment of homology theory. Although the emphasis is on computation, indeed the matrix reduction algorithm is covered, most of the standard fare is discussed: relative homology, exact sequences, and cohomology. Duality theorems: Poincaré, Lefschetz, and Alexander, are also discussed; and the last chapter of this part of the text covers Morse Theory. The purpose of the discussion is to convey the main ideas, rather than a complete understanding of the details; as with the first part, most results are stated without proof. The third and final part of the text discusses persistent homology and its applications. Persistent homology is a fairly recent discovery: the original paper was published in 2002, and provides a convenient graphical representation, called a persistence diagram, of the homological information (essentially the Betti numbers) present in a filtration of a topological space. The first chapter in this part of the text discusses the persistent homology of a simplicial complex filtered by ordering its simplices: a data structure and algorithm used for efficient computation, as well as the even more recent (2009) notion of extended persistence of a simplicial manifold. The penultimate chapter discusses how persistence diagrams are affected by changing the ordering of the simplices. The book concludes with a chapter on applications to molecular biology: gene expression and protein docking, as well as applications to biological imaging: cell segmentation and detecting root architecture. The text is well-written and well-organized, with only a few typographical errors scattered throughout. A broad range of topics are covered, although there is an understandable bias towards topics that reflect the authors' own interests; indeed some topics in computational topology, most notably Forman's discrete version of Morse Theory, are not mentioned. While the book is intended as a textbook for graduates and advanced undergraduates in mathematics or computer science, the expert is also likely to find some of the discussions of interest. The exposition is often terse, potentially requiring the less experienced reader to consult other references on some topics; however, the authors provide bibliographic notes at the end of each section. Exercises are also provided at the end of each chapter. *Jason Hanson (Redmond)*

Hacon, Christopher Derek

Spellings: Hacon, Christopher D. [32] Hacon, Christopher [3] Hacon, C. [3] Hacon, Christopher Derek [2]
Author-Id: hacon.christopher-derek
Publications: 40 including 1 Book(s) and 35 Journal Article(s)

MSC 2010

40	**14** · Algebraic geometry
1	**11** · Number theory
1	**16** · Associative rings and algebras
1	**17** · Nonassociative rings and algebras
1	**18** · Category theory, homological algebra

more ...

Journals

4	Inventiones Mathematicae
4	Journal für die Reine und Angewandte Mathematik
2	Compositio Mathematica
2	Duke Mathematical Journal
2	Journal of Algebra

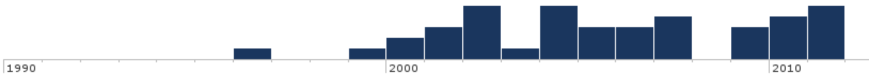
more ...

Co-Authors

12	Chen, Jungkai Alfred
6	McKernan, James
3	Pardini, Rita
2	de Fernex, Tommaso
2	Fioresi, Rita

more ...

Publication Years



Zbl 1210.14019
Birkar, Caucher; Cascini, Paolo;
Hacon, Christopher D.; McKernan, James
Existence of minimal models for varieties of log general type.
J. Am. Math. Soc. 23, No. 2, 405-468 (2010).

The paper under review is a milestone in the birational and biregular geometry of higher dimensional algebraic varieties. The main result is the existence of a log terminal model for log terminal pairs (X, Δ), with big boundary, Δ, and pseudo-effective adjoint divisor, $K_X + \Delta$.
The Minimal Model Program aims to show that given any n-fold X there is a variety Y, birational to X such that either the canonical class K_Y is nef or Y admits a fibration with ample anticanonical class. The two main lines to pursued this task are prove the finite generation of the canonical ring and kill the negative part of the canonical class via birational transformations. One of the main novelty of this paper is a way to combine both approach. Instead of trying to explain and write in details the main technical result, obtained via a complicate induction, I think it is much more profitable to list some of the outstanding consequences of it.
Any smooth variety of general type has a minimal model, a canonical model a model with Kähler–Einstein metric, and the canonical ring is finitely gen-

erated. Any klt pair (X, Δ) has the canonical ring finitely generated. Fano manifolds are Mori dream spaces [see *Y. Hu* and *S. Keel*, Mich. Math. J. 48, Spec. Vol., 331–348 (2000; Zbl 1077.14554)]. Let (X, Δ) be a klt pair. Then flips for (X, Δ) exist. If $K_X + \Delta$ is not pseudo-effective then X is birational to a Mori fiber space. It is proven the Inversion of adjunction for arbitrary log pairs. This together with applications to moduli spaces, birational geometry and biregular geometry. *Massimiliano Mella (Ferrara)*

Zbl 1210.14021
Hacon, Christopher D.; McKernan, James
Existence of minimal models for varieties of log general type. II.
J. Am. Math. Soc. 23, No. 2, 469-490 (2010).

The paper under review is the natural completion of *C. Birkar, P. Cascini, C. Hacon* and *J. McKernan* [J. Am. Math. Soc. 23, No. 2, 405–468 (2010; Zbl 5775673)]. Here the finite generation of the canonical ring for an arbitrary smooth projective manifold is proven, removing the assumptions of general type. The result concludes the Holy Grail search of Minimal Models for projective algebraic varieties and it is a fundamental result for the birational and biregular theory of projective algebraic varieties. The proof is via induction on the dimension and relies on proving the existence of a special class of flipping contractions assuming finite generation of canonical algebras in one dimension less. *Massimiliano Mella (Ferrara)*

Zbl 1232.14008
Hacon, Christopher D.; McKernan, James
Boundedness results in birational geometry.
Bhatia, Rajendra (ed.) et al., Proceedings of the international congress of mathematicians (ICM 2010), Hyderabad, India, August 19–27, 2010. Vol. II: Invited lectures. Hackensack, NJ: World Scientific; New Delhi: Hindustan Book Agency (ISBN 978-981-4324-32-8/hbk; 978-81-85931-08-3/hbk; 978-981-4324-30-4/set; 978-981-4324-35-9/ebook). 427-449 (2011).

The paper under review is a very well written survey about recent progress related to boundedness results in higher dimensional birational geometry. Namely, if X is an n-dimensional smooth complex projective variety of general type, it is a natural and important question to determine existence (and explicit value) of a positive constant r_n which depends only on n such that, for every $r \geq r_n$, the pluri-canonical map $\phi_r : X \dashrightarrow \mathbb{P}H^0(X, rK_X)$ is birational. This question has many consequences, the most important one probably being that it is an essential building block in the construction of a moduli space of canonically polarised varieties of general type.
The existence of such a constant r_1 is classical and it is known that $r_1 = 3$. For surfaces, it was proved that $r_2 = 5$ in [*E. Bombieri*, Several Complex Variables I, Conf. Univ. Maryland 1970, 35–87 (1970; Zbl 213.47601)]. Finally, the proof of existence for every n was given in [*C. D. Hacon* and J. McKernan, Invent. Math. 166, No. 1, 1–25 (2006; Zbl 1121.14011)], [*S. Takayama*, Invent. Math.

165, No. 3, 551–587 (2006; Zbl 1108.14031)] and [*H. Tsuji*, Osaka J. Math. 44, No. 3, 723–764 (2007; Zbl 1186.14043)]. The authors of the paper under review give an instructive sketch of the proof. It was shown in [*J. A. Chen* and *M. Chen*, J. Differ. Geom. 86, No. 2, 237–271 (2010; Zbl 1218.14026)] that $r_3 \leq 73$. The paper also deals with related topics, such as the analogous result in the case of varieties not of general type or in the case of log general type, and also boundedness of the automorphism group of varieties of general type. One of the main contributions of the paper is that it presents clearly and coherently the ideas related to boundedness. With difficult topics such as this one, it is important for the coming generations of mathematicians that the techniques and the behind-the-scenes philosophy are demystified, and this survey is particularly important from that point of view. *Vladimir Lazic (Bayreuth)*

Kazhdan, David A.

Spellings: Kazhdan, David [59] Kazhdan, D. [27] Kazhdan, D.A. [18] Kazhdan, David A. [5]
Author-Id: kazhdan.david-a
Publications: 109 including 2 Book(s) and 73 Journal Article(s)

MSC 2010

52	**22** · Topological groups, Lie groups
34	**11** · Number theory
27	**17** · Nonassociative rings and algebras
20	**14** · Algebraic geometry
18	**20** · Group theory and generalizations

more ...

Journals

9	Selecta Mathematica. New Series
6	Functional Analysis and its Applications
6	Journal d'Analyse Mathématique
5	Advances in Mathematics
5	Geometric and Functional Analysis - GAFA

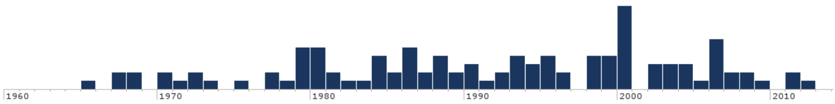
more ...

Co-Authors

10	Lusztig, George
8	Etingof, Pavel I.
5	Braverman, Alexander
5	Flicker, Yuval Z.
4	Deligne, Pierre

more ...

Publication Years



Zbl 1235.22027
Braverman, Alexander; Kazhdan, David
The spherical Hecke algebra for affine Kac-Moody groups. I.
Ann. Math. (2) 174, No. 3, 1603-1642 (2011).

Let F be a global field and $\mathbb{A}_F$ its ring of adeles. For a split reductive group G over F, the classical Langlands duality predicts that irreducible automorphic representations of $G(\mathbb{A}_F)$ are closely related to homomorphisms from the absolute Galois group Gal_F of F to the Langlands dual group $\check{G}$. Similarly, if G is a split reductive group over a local non-archimedean field K, then Langlands duality predicts a relation between irreducible representations of $G(K)$ and homomorphisms from Gal_K to $\check{G}$.
Let O be the ring of integers in K. The starting point for Langlands duality is the so-called *Satake isomorphism* relating the spherical Hecke algebra of $G(O)$-biinvariant compactly supported $\mathbb{C}$-valued measures on $G(K)$ and the complexified Grothendieck ring of the category of finite dimensional representations of $\check{G}$.
The global project of the authors is to develop some sort of Langlands theory in the case when G is replaced by an affine Kac-Moody group. In this paper the authors define the spherical Hecke algebra for an untwisted affine Kac-Moody group over a local non-archimedean field and prove a generalization of the Satake isomorphism for this algebra, relating it to integrable representations of the Langlands dual affine Kac-Moody group.

Volodymyr Mazorchuk (Uppsala)

Łuczak, Tomasz

Spellings: Łuczak, Tomasz [108] Łuczak, T. [19] Łuczak, Tomasz [13] Luczak, Tomasz [6] Łuczak, Tomasz [1]
Author-Id: luczak.tomasz
Publications: 147 including 3 Book(s) and 126 Journal Article(s)

MSC 2010

124	**05** · Combinatorics
21	**60** · Probability theory and stochastic processes
19	**11** · Number theory
10	**68** · Computer science
5	**03** · Mathematical logic

more ...

Journals

19	Random Structures & Algorithms
15	Discrete Mathematics
11	Combinatorics, Probability and Computing
10	Journal of Combinatorial Theory. Series B
9	Journal of Graph Theory

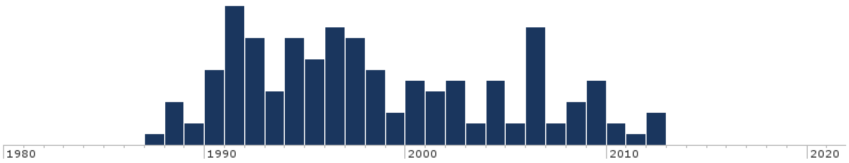
more ...

Co-Authors

13	Rodl, Vojtech
12	Schoen, Tomasz
11	Kohayakawa, Yoshiharu
11	Ruciński, Andrzej
8	Haxell, Penny E.

more ...

Publication Years



Zbl 962.11013
Łuczak, Tomasz; Schoen, Tomasz
On the maximal density of sum-free sets.
Acta Arith. 95, No. 3, 225-229 (2000).

For a set $A \subseteq \mathbb{N}$ the subset sums of A are defined by $P(A) = \{\sum_{a \in B} a : B \subseteq A, 1 \leq |B| < \infty\}$. Furthermore let $Q(A) = \{\sum_{a \in B} a : B \subseteq A, 2 \leq |B| < \infty\}$. A set A is said to be sum-free if $A \cap Q(A) = \emptyset$. Let $A(n)$ be the counting function of A, i.e. $A(n) = |A \cap \{1, 2, \ldots, n\}|$.
In the present paper the authors prove the following results: If A is a set of natural numbers with $A(n) > 402\sqrt{n \cdot \log n}$ then $P(A)$ contains an infinite arithmetic progression.
This result is an improvement of a well-known result of Folkman (and also proved by the reviewer). Furthermore the authors also improve a result of Deshouillers, Erdős and Melfi, proving if $A \subseteq \mathbb{N}$ is a sum-free set then for large n $A(n) \leq 403\sqrt{n \cdot \log n}$. On the other hand it is proved there exists a sum-free set B such that $B(n) \geq n^{1/2} / \log n^{1/2+\varepsilon}$. *Norbert Hegyvári (Budapest).*

Zbl 968.05003
Janson, Svante; Łuczak, Tomasz; Ruciński, Andrzej
Random graphs.
Wiley-Interscience Series in Discrete Mathematics and Optimization.
New York, NY: Wiley, (ISBN 0-471-17541-2/hbk). xii, 333 p. £ 53.95 (2000).

This book is a beautiful presentation of new developments in the asymptotic theory of random graphs. It covers the period since about 1985 when Bollobás' monograph with the same title appeared (see Zbl 567.05042). It emphasizes new techniques and tools that have successfully contributed to recent progress for Bernoulli graphs and uniform random graphs. Among the topics studied are thresholds for monotone properties of random subsets, exponential bounds for tail probabilities, thresholds for subgraph containment, perfect matchings, the emergence of the giant component, convergence in distribution, approximation by projection, vertex coloring and the chromatic number, edge coloring and monochromatic triangles, and random regular graphs. The last chapter gives results for families of graph properties by employing notions and facts from mathematical logic. *Ove Frank (Stockholm).*

Zbl 1167.11008
Łuczak, Tomasz; Schoen, Tomasz
On a problem of Konyagin.
Acta Arith. 134, No. 2, 101-109 (2008).

Let $(G,+)$ be an abelian group. For $A \subseteq G$ and $t \in G$, let $\nu(t) = |\{(a,b) \in A\times A \,:\, t = a+b\}|$. Let $\nu(A) = \min_{t\in A+A} \nu(t)$. Sergei V. Konyagin [see *V. F. Lev*, "Reconstructing integer sets from their representation functions", Electron. J. Comb. 11, No. 1, Research paper R78, 6 p., electronic only (2004; Zbl 1068.-11006)], asked the following question:
"Do there exist constants $\varepsilon, C > 0$ such that for every sufficiently large prime p and each set $A \subseteq \mathbb{Z}/p\mathbb{Z}$ with $|A| < \sqrt{p}$, we have $\nu(A) \le C|A|^{1-\varepsilon}$?"
In the paper under review the authors prove a result towards this direction. The main theorem states that there are positive constants C_1, C_2 such that if $A \subseteq \mathbb{Z}/p\mathbb{Z}$ verifies $|A| < C_1 p^{2^{-d-1}}$, d integer, $d \ge 3$ then

$$\nu(A) \le C_2|A|.$$

More precise, but technically complicated, information connecting C_1, C_2 and d is given.
Ingredients of the proof are Dirichlet's approximation theorem and the following result of Plünnecke and Ruzsa:
"Let C, D be finite subsets of an abelian group. If $|C + D| \le K|D|$, then for every $k \ge 1$,

$$|kC| \le K^k|D|.$$

Georges Grekos (St Etienne)

Constantin, Adrian

Spellings: Constantin, Adrian [133] Constantin, A. [15]
Author-Id: constantin.adrian
Publications: 148 including 1 Book(s) and 142 Journal Article(s)

MSC 2010

80	**35** · Partial differential equations (PDE)
53	**76** · Fluid mechanics
31	**34** · Ordinary differential equations (ODE)
22	**37** · Dynamical systems and ergodic theory
11	**60** · Probability theory and stochastic processes

more ...

Journals

6	Communications on Pure and Applied Mathematics
6	Journal of Mathematical Analysis and Applications
4	Analele Universităţii din Timişoara. Seria Ştiinţe Matematice
4	Archive for Rational Mechanics and Analysis
4	Physics Letters. A

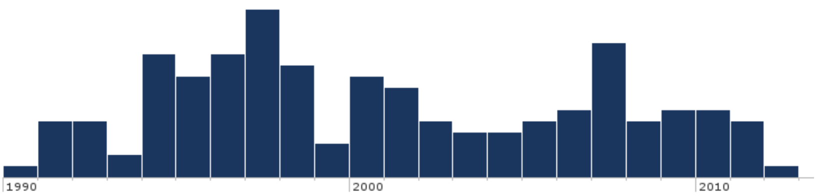
more ...

Co-Authors

16	Escher, Joachim
11	Strauss, Walter Alexander
7	Ivanov, Rossen I.
6	Kolev, Boris
4	Johnson, Robin S.

more ...

Publication Years

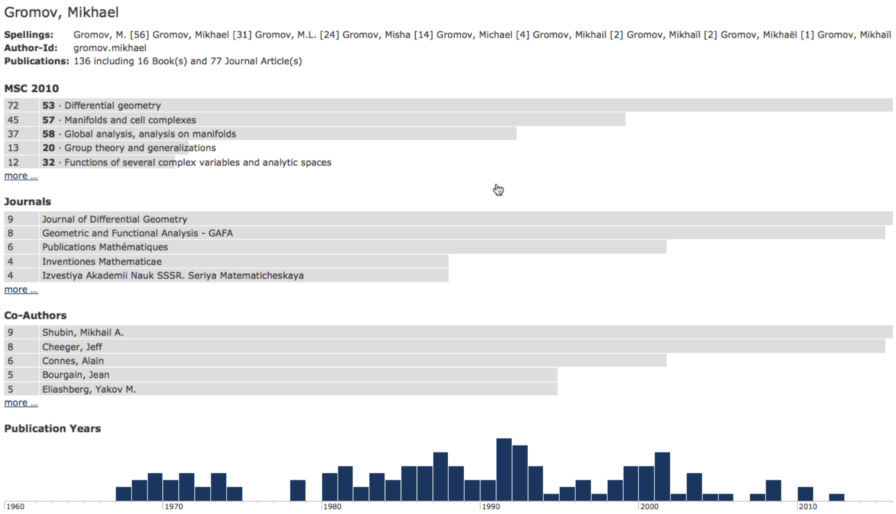


Zbl 1229.35203
Constantin, Adrian; Varvaruca, Eugen
Steady periodic water waves with constant vorticity: Regularity and local bifurcation.
Arch. Ration. Mech. Anal. 199, No. 1, 33-67 (2011).

Authors' abstract: "This paper studies periodic traveling gravity waves at the free surface of water in a flow of constant vorticity over a flat bed. Using conformal mappings the free-boundary problem is transformed into a quasi-linear pseudodifferential equation for a periodic function of one variable. The new formulation leads to a regularity result and, by use of bifurcation theory, to the existence of waves of small amplitude even in the presence of stagnation points in the flow." *A. D. Osborne (Keele)*

Zbl 1228.35076
Constantin, Adrian; Escher, Joachim
Analyticity of periodic traveling free surface water waves with vorticity.
Ann. Math. (2) 173, No. 1, 559-568 (2011).

Summary: We prove that the profile of a periodic traveling wave propagating at the surface of water above a flat bed in a flow with a real analytic vorticity must be real analytic, provided the wave speed exceeds the horizontal fluid velocity throughout the flow. The real analyticity of each streamline beneath the free surface holds even if the vorticity is only Hölder continuously differentiable.



Zbl 1211.92019
Gromov, Misha
Crystals, proteins, stability and isoperimetry.
Bull. Am. Math. Soc., New Ser. 48, No. 2, 229-257 (2011).

Summary: We attempt to formulate several mathematical problems suggested by structural patterns present in biomolecular assemblies. Our description of these patterns, by necessity brief and over-concentrated in some places, is self-contained, albeit on a superficial level. An attentive reader is likely to stumble upon a cryptic line here and there; however, things will become more transparent at a second reading and/or at a later point in the article.

Serfaty, Sylvia

Spellings: Serfaty, Sylvia [43] Serfaty, S. [3]
Author-Id: serfaty.sylvia
Publications: 45 including 1 Book(s) and 33 Journal Article(s)

MSC 2010

40	**35** · Partial differential equations (PDE)
30	**82** · Statistical mechanics, structure of matter
20	**49** · Calculus of variations and optimal control; optimization
12	**58** · Global analysis, analysis on manifolds
4	**91** · Game theory, economics, social and behavioral sciences

more ...

Journals

5	Communications on Pure and Applied Mathematics
3	Communications in Contemporary Mathematics
3	European Series in Applied and Industrial Mathematics (ESAIM): Control, Optimization and Calculus of Variations
2	Annales de l'Institut Henri Poincaré. Analyse Non Linéaire
2	Discrete and Continuous Dynamical Systems

more ...

Co-Authors

14	Sandier, Etienne
3	Kohn, Robert V.
3	Rivière, Tristan
2	Aftalion, Amandine
2	Ambrosio, Luigi

more ...

Publication Years

Zbl 1112.35002
Sandier, Etienne; Serfaty, Sylvia
Vortices in the magnetic Ginzburg-Landau model.
Progress in Nonlinear Differential Equations and Their Applications 70.
Basel: Birkhäuser (ISBN 978-0-8176-4316-4/hbk; 978-0-8176-4550-2/e-book). xii, 322 p. EUR 98.00/net; SFR 158.00 (2007).

This book deals with the mathematical study of the two-dimensional Ginzburg-Landau model with magnetic field. This important model was introduced by Ginzburg and Landau in the 1950s as a phenomenological model to describe superconductivity consisting in then complete loss of resistivity of certain metals and alloys at very low temperatures. The Ginzburg-Landau model allows to predict the possibility of a mixed state in type II superconductors where triangular lattices appear. These vortices can be described as a quantized amount of vorticity of the superconducting current localized near a point.
Chapter 1 presents the plan of the book. The goal of the authors consists in describing, through a mathematical analysis, in the asymptotic limit (e small), the minimizers of the Gibbs energy Ge and their critical points in terms of their vortices. This includes the determination of their precise optimal vortex-locations.

Chapter 2 starts with an heuristic presentation of the model.
Chapter 3 describes the basic mathematical results on the Ginzburg-Landau equation (existence of solutions, a priori estimates,. . .).
Chapter 4 presents the "ball construction method" allowing to obtain universal lower bounds for Ginzburg-Landau energies in terms of cortices and their degrees.
The second part of the book (chapters 7 through 12) presents results obtained through minimization and the third part (chapter 13) contains results for nonminimizing solutions.
All parts of this interesting book are clearly and rigorously written. A consistent bibliography is given and several open problems are detailed. This work has to be recommended. *Yves Cherruault (Paris)*

Zbl 1138.82034
Aftalion, Amandine; Serfaty, Sylvia
Lowest Landau level approach in superconductivity for the Abrikosov lattice close to H_{c_2}.
Sel. Math., New Ser. 13, No. 2, 183-202 (2007).

This paper deals with the asymptotic study of the Ginzburg-Landau energy with applied magnetic field h_{ex}. The authors are interested in the regime where h_{ex} behaves like ε^{-2}, which characterizes its behavior near the second critical field. This relies on the study of the associated energy functional on a reduced space, which is the first eigenspace for a magnetic operator, also called the lowest Landau level. The analysis is based on refined elliptic estimates combined with a careful asymptotic analysis. Several open problems are also raised in the present paper. *Vicenţiu D. Rădulescu (Craiova)*

Zbl 1159.26004
Serfaty, Sylvia; Tice, Ian
Lorentz space estimates for the Ginzburg-Landau energy.
J. Funct. Anal. 254, No. 3, 773-825 (2008).

The authors consider the functional representing the Ginzburg-Landau free energy of the model of superconductivity, that is

$$F_\varepsilon(u,A) = \frac{1}{2}\int_\Omega |\nabla_A u|^2| + |\mathrm{curl} A|^2 + \frac{(1-|u|^2)^2}{2\varepsilon^2},$$

where Ω is a bounded regular domain in $\mathbb{R}^2$, u is a complex-valued function and $A \in \mathbb{R}^2$ is a vector field in Ω, representing the vector potential of the magnetic field. The authors focus on the regime for small ε. Note that ε is a material constants whose smallness indicates the superconductivity of the so-called type II. In this setting, u can have vortices (zeros of non-zero topological degree). The authors also consider the case without magnetic field ($A = 0$). The main objective is to obtain lower bounds for F_ε, in particular, bounds that relate $F_\varepsilon(u,A)$ to the norms of ∇u in the Marcinkiewicz space

$L^{2,\infty}$. These spaces and their dual Lorentz spaces $L^{2,1}$ have been studied in a similar context by *F.-H. Lin* and *T. Rivière* [Commun. Pure Appl. Math. 54, No. 2, 206–228 (2001; Zbl 1033.58013)]. Usually, the Ginzburg–Landau energy is unbounded as ε tends to zero, in fact, it blows up roughly like $\pi n|\log n|\varepsilon$, where n is the number of vortices.
The authors present several main results. First, they improve the L^2–lower bounds obtained by *E. Sandier* and *S. Serfaty* [Vortices in the magnetic Ginzburg-Landau model. Basel: Birkhäuser (2007; Zbl 1112.35002)] by a ball construction as follows:

$$\frac{1}{2}\int_V |\nabla_A u|^2 + \frac{1}{2\varepsilon^2}(1-|u|^2)^2 + r^2(\mathrm{curl}A)^2 \geq \pi D(\log\frac{r}{\varepsilon D} - C) + \frac{1}{18}\int |\nabla_{A+G}u|^2 + \frac{1}{2\varepsilon^2}(1-|u|^2)^2,$$

where V is the intersection of the set $\{x \in \Omega,\ \mathrm{dist}(x, \partial\Omega) > \varepsilon\}$ with a union of a certain disjoint collection of closed balls B, $D = \sum_{B\subset\Omega}|d_B|$, and $d_B = \deg(u, B)$. The improvement consists in the addition of the term $\frac{1}{18}\int|\nabla_{A+G}u|^2$.
Next, in their principal result, the authors give a bound involving the Marcinkiewicz norm, precisely

$$\frac{1}{2}\int_V |\nabla_A u|^2 + \frac{1}{2\varepsilon^2}(1-|u|^2)^2 + r^2(\mathrm{curl}A)^2 + \pi\sum|d_B|^2 \geq C\|\nabla_A\|^2_{L^{2,\infty}(V)} + \pi\sum|d_B|(\log\frac{r}{\varepsilon\sum|d_B|} - C),$$

where the sum is taken over all balls B in the above-mentioned collection. Various consequences are pointed out and a sequel paper in which the ideas are extended to an applied magnetic field is announced. *Luboš Pick (Praha)*

Shelah, Saharon

Spellings: Shelah, Saharon [841] Shelah, S. [66]
Author-Id: shelah.saharon
Publications: 906 including 10 Book(s) and 816 Journal Article(s)

MSC 2010

808	**03** · Mathematical logic
153	**20** · Group theory and generalizations
82	**06** · Ordered structures
75	**05** · Combinatorics
68	**54** · General topology

more ...

Journals

134	Israel Journal of Mathematics
132	The Journal of Symbolic Logic
75	Annals of Pure and Applied Logic
53	Fundamenta Mathematicae
45	Archive for Mathematical Logic

more ...

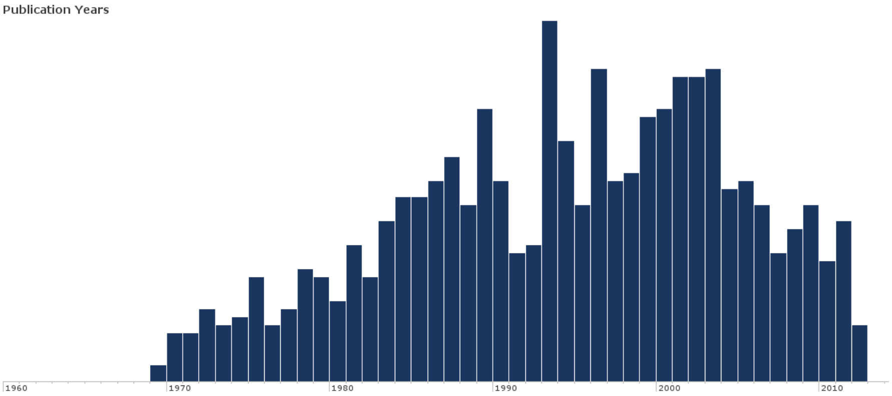
Co-Authors

32	Göbel, Rüdiger
27	Rosłanowski, Andrzej
26	Mekler, Alan H.
22	Eklof, Paul C.
22	Gurevich, Yuri

more ...

Publication Years

Publication Years



Zbl 1225.03036
Shelah, Saharon
Classification theory for abstract elementary classes.
Studies in Logic (London) 18. Mathematical Logic and Foundations.
London: College Publications (ISBN 978-1-904987-71-0/pbk).
vii, 813 p. (2009).

Abstract Elementary Classes (AEC) are the focus of a two-volume work by Saharon Shelah, the first volume of which is under review here (for Volume II see the review below). AEC provide the setting in which Shelah seeks to extend his model-theoretic classification theory beyond first-order theories, for example, to classes determined by sentences of $L_{\omega_1,\omega}$ and $L_{\omega_1,\omega}(Q)$, where Q is the quantifier “uncountably many". He desires a broad generalization of his earlier theory, one in which he can define concepts that provide “dividing lines” between structure theorems and non-structure theorems (like the concepts of stability and superstability that proved to be so successful for first-order theories).

The main body of Volume I consists of four chapters: (I) Abstract elementary classes near $\aleph_1$; (II) Categoricity in abstract elementary classes: going up inductively; (III) Toward classification theory of good λ-frames and abstract elementary classes; (IV) Categoricity and solvability of a.e.c., quite highly. The chapters are (mostly) independent articles, having their own introductions.

The volume itself features two introductory sections: one for model theorists and one for "the logically challenged". In fact, one can say that Shelah has aimed his text at this latter group; he wants to distance himself as much as possible from logical jargon in order to reach a wider mathematical audience. A particularly useful introductory section is the "Annotated Contents". In it, each chapter's sections are listed along with a brief description of the particular section's contents and the author's intended goal. (The chapters of Volume II also appear here.) After the overall introduction, the Annotated Contents make an excellent starting point for the prospective user of this volume. *J. M. Plotkin (East Lansing)*

Talagrand, Michel

Spellings: Talagrand, Michel [267] Talagrand, M. [30]
Author-Id: talagrand.michel
Publications: 297 including 7 Book(s) and 252 Journal Article(s)

MSC 2010

156	**60** · Probability theory and stochastic processes
98	**46** · Functional analysis
74	**28** · Measure and integration
42	**82** · Statistical mechanics, structure of matter
23	**54** · General topology

more ...

Journals

37	The Annals of Probability
18	Israel Journal of Mathematics
17	Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences, Série A
17	Probability Theory and Related Fields
13	Proceedings of the American Mathematical Society

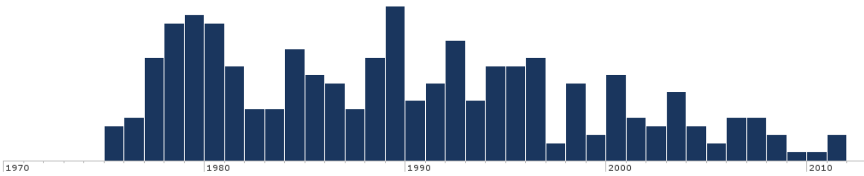
more ...

Co-Authors

31	Rhee, Wansoo T.
8	Ledoux, Michel
7	Fremlin, David H.
3	Bourgain, Jean
3	Ghoussoub, Nassif A.

more ...

Publication Years



Zbl 748.60004
Ledoux, Michel; Talagrand, Michel
Probability in Banach spaces. Isoperimetry and processes. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge, 23.
Berlin etc.: Springer-Verlag. xii, 480 p. (1991).

As the authors state, "this book tries to present some of the main aspects of the theory of probability in Banach spaces, from the foundation of the topic to the latest developments and current research questions". The authors have succeeded admirably; the book sums up and discusses most of the important results of probability theory in Banach spaces which have been discovered in the last two decades. Probability in Banach spaces is a branch of mathematics that lies in a joint of classical probability, measure theory and functional analysis; it studies properties of random variables taking values in a Banach space, the behaviour of their distributions, limit theorems etc. These properties as well as statements of the limit theorems are essentially dependent on the geometry of the Banach space, and this fact stipulates the nature of the theory, where the methods of classical probability interlace with those of measure theory, geometry of a Banach space and with abstract analytic methods.

This very comprehensive book develops a wide variety of the methods existing nowadays in probability in Banach spaces. However the authors

select among them and focus the reader's attention on the isoperimetric inequalities method, concentration of measure phenomenon, and the abstract random processes technique. This is reflected in the subtitle of the book: Isoperimetry and processes. The authors themselves contributed essentially to the elaboration of these methods. The use of isoperimetric inequalities to the concentration inequalities, tail estimates and integrability theorems of probability in Banach spaces has led today to rather a complete picture of the theory. The second part of the subtitle – Processes – is related to a big chapter of probability in Banach spaces to which the second author has contributed very much. Here the central issue is the celebrated Fernique-Talagrand theorem solving the problem on continuity and boundedness of samples of a Gaussian process by means of the majorizing measure technique. This material is naturally included into the framework of probability in Banach spaces, since for instance, a random process with continuous sample paths can be considered as a random variable taking values in the Banach space of continuous functions.
It seems to use that the monograph will become an event for mathematicians working or interested in probability in Banach spaces. The table of contents we bring below shows that the monograph covers most of the main questions of the theory. The authors did not intend to cover all the aspects of the theory. Among the topics not covered they mention Banach space valued martingales, infinitely divisible distributions, rate of convergence in the central limit theorem. We would also add characteristic functionals, their topological descriptions, cylindrical measures and Radonifying operators. The book is equipped with an extensive bibliography.
Table of contents: Chapter 1. Isoperimetric inequalities and the concentration of measure phenomenon; Chapter 2. Generalities on Banach space valued random variables and random processes; Chapter 3. Gaussian random variables; Chapter 4. Rademacher averages; Chapter 5. Stable random variables; Chapter 6. Sums of independent random variables; Chapter 7. The strong law of large numbers; Chapter 8. The law of iterated logarithm; Chapter 9. Type and cotype of Banach spaces; Chapter 10. The central limit theorem; Chapter 11. Regularity of random processes; Chapter 12. Regularity of Gaussian and stable processes; Chapter 13. Stationary processes and random Fourier series; Chapter 14. Empirical process methods in probability in Banach spaces; Chapter 15. Applications to Banach space theory. *S. A. Chobanjan (Tbilisi)*

Zbl 1075.60001
Talagrand, Michel
The generic chaining. Upper and lower bounds of stochastic processes.
Springer Monographs in Mathematics.
Berlin: Springer (ISBN 3-540-24518-9/hbk). viii, 222 p.
EUR 79.95/net; £ 61.50; sFr 135.50; $ 99.00 (2005).

This textbook gives a systematic approach to the problem of deriving good bounds for stochastic processes using the "generic chaining" method, a variation of the classical chaining arguments that go back to Kolmogorov. More precisely, the problem consists of finding bounds on $E(\sup_{t\in T} X_t)$ for a stochastic process $(X_t)_{t\in T}$, depending on and exploiting the structure of the index set T, which is always assumed to be a metric space. After essential contributions by R. Dudley and X. Fernique, the question of characterizing boundedness of Gaussian processes was solved by the author in 1985, when he proved a conjecture due to Fernique with the aid of majorizing measures. This result and extensions thereof have been reworked in recent years by the author by systematically exploiting the generic chaining technique, thus replacing the rather technical and difficult tool of majorizing measures and putting the topic on new ground. The presentation in this book follows this rework and takes the reader from a simple and intuitive explanantion of the basic idea underlying the chaining technique to the edge of todays knowledge. On this way, the reader is introduced to a number of open problems in the field, and for the solution to some of them the author has advertised a cash reward. The entertaining and sometimes humorous style makes this book a pleasure to read.

The material in the book is organized along the following lines. After introducing the reader to the idea of generic chaining and partitioning schemes (Chapter 1), Chapter 2 is devoted to the study of Gaussian processes, and structures and processes related to them (e.g. stable processes, Gaussian chaos). Chapter 3 deals with so-called matching theorems, which deal with the following question. Given a (finite) sequence $X_1, \ldots, X_N$ of i.i.d. random variables, uniformly distributed over the unit square $[0,1]^2$, how far are these points from being evenly spread over the square. This is measured via different notions for the cost of matching the set of random points against a deterministic set of points which is (in some reasonable sense) evenly spread. In Chapter 4 the Bernoulli conjecture is treated, which states a bound for $E(\sup_{t\in T} X_t)$ in the case when all X_t are in the closure of an i.i.d. Bernoulli sequence. The author presents some considerations towards a proof of the conjecture, but also stresses that this presentation is far from a solution (proof) of the conjecture. The last two chapters (5 and 6) deal with technical extensions and special applications. In Chapter 5 the partitioning scheme of the chaining method is generalized to the case when the metric space T is endowed with an entire family of distances that are required to control various aspects of X, which is then applied in order to derive bounds for infinitely divisible processes. Finally, in Chapter 6 some applications to Banach space theory are presented. *Dierk Peithmann (Berlin)*

Brendle, Simon

Spellings: Brendle, Simon [41] Brendle, S. [2]
Author-Id: brendle.simon
Publications: 43 including 2 Book(s) and 34 Journal Article(s)

MSC 2010

37	**53** · Differential geometry
9	**35** · Partial differential equations (PDE)
5	**58** · Global analysis, analysis on manifolds
4	**47** · Operator theory
2	**91** · Game theory, economics, social and behavioral sciences

more ...

Journals

6	Journal of Differential Geometry
3	Duke Mathematical Journal
2	Communications in Analysis and Geometry
2	Communications on Pure and Applied Mathematics
2	Inventiones Mathematicae

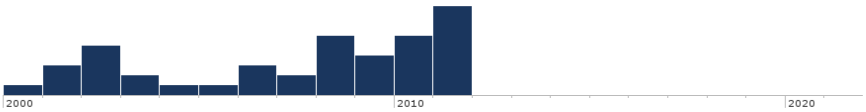
more ...

Co-Authors

6	Schoen, Richard M.
3	Nagel, Rainer J.
2	Marques, Fernando Coda
2	Neves, André
1	Blake, Mark D.

more ...

Publication Years



Zbl 1209.53054
Brendle, Simon
A note on Ricci flow and optimal transportation.
J. Eur. Math. Soc. (JEMS) 13, No. 1, 249-258 (2011).

This paper deals with an extension of the following classical interpolation inequality due to *A. Prekopa* [Acta Sci. Math. 32, 301–316 (1971; Zbl 0235.90044)] and *L. Leindler* [Proc. Conf. Oberwolfach 1971, ISNM 20, 182–184 (1972; Zbl 0256.26015)].
Theorem 1. Fix a real number $0 < \lambda < 1$. Let $u_1, u_2, v : \mathbb{R}^n \to \mathbb{R}$ be nonnegative measurable functions satisfying

$$v\,(\,(1-\lambda)x + \lambda y\,) \geq u_1(x)^{1-\lambda} u_2(y)^{\lambda}$$

for all points $x, y \in \mathbb{R}^n$. Then

$$\int_{\mathbb{R}^n} v \geq \left(\int_{\mathbb{R}^n} u_1 \right)^{1-\lambda} \left(\int_{\mathbb{R}^n} u_2 \right)^{\lambda}.$$

Some generalizations in the context of Riemannian geometry were obtained in [*D. Cordero-Erausquin, R. McCann* and *M. Schmuckenschläger*, Invent. Math. 146, No. 2, 219–257 (2001; Zbl 1026.58018); Ann. Fac. Sci. Toulouse, Math.

(6) 15, No. 4, 613–635 (2006; Zbl 1125.58007)]. The aim of the paper is to obtain other generalizations of the interpolation inequality replacing the Riemannian distance by the $\mathcal{L}$-distance of Perelman.
Namely, let M be a compact n-dimensional manifold, and let $g(t)$, $t \in [0,T]$ be a one-parameter family of metrics on M, which evolve by the backward Ricci flow, i.e. $\frac{\partial g(t)}{\partial t} = 2\mathrm{Ric}_{g(t)}$. The $\mathcal{L}$-length of a path $\gamma : [\tau_1, \tau_2] \to M$ is defined by

$$\mathcal{L}(\gamma) = \int_{\tau_1} \tau_2 \sqrt{t} \left(R_{g(t)}(\gamma(t)) + |\gamma'(t)|^2_{g(t)} \right) dt, \qquad (*)$$

where $R_{g(t)}$ stands for the scalar curvature of $g(t)$. The corresponding $\mathcal{L}$-distance is defined by

$$Q(x, \tau_1; y, \tau_2) = \inf\Big\{ \mathcal{L}(\gamma) | \gamma : [\tau_1, \tau_2] \to M, \gamma(\tau_1) = x, \gamma(\tau_2) = y \Big\}.$$

The main results of the paper is the following extension of the interpolation inequality in the setting of Ricci flow and $\mathcal{L}$-distance.
Theorem 2. Fix real numbers τ_1, τ_2, τ such that $0 < \tau_1 < \tau < \tau_2 < T$. Write $\frac{1}{\sqrt{\tau}} = \frac{1-\lambda}{\sqrt{\tau_1}} + \frac{\lambda}{\sqrt{\tau_2}}$, where $0 < \lambda < 1$. Let u_1, u_2, $v : M \to \mathbb{R}$ be nonnegative measurable functions such that

$$\left(\frac{\tau}{\tau_1^{1-\lambda} \tau_2^{\lambda}} \right)^{n/2} v(\gamma(\tau)) \geq \exp\left(-\frac{1-\lambda}{2\sqrt{\tau_1}} Q(\gamma(\tau_1), \tau_1; \gamma(\tau), \tau) \right) u_1(\gamma(\tau_1))^{1-\lambda}$$
$$\cdot \exp\left(\frac{\lambda}{2\sqrt{\tau_2}} Q(\gamma(\tau), \tau; \gamma(\tau_2), \tau_2) \right) u_2(\gamma(\tau_2))^{\lambda}$$

for every minimizing $\mathcal{L}$-geodesic $\gamma : [\tau_1, \tau_2] \to M$. Then

$$\int_M v \, \mathrm{dvol}_{g(\tau)} \geq \left(\int_M u_1 \, \mathrm{dvol}_{g(\tau_1)} \right)^{1-\lambda} \left(\int_M u_2 \, \mathrm{dvol}_{g(\tau_2)} \right)^{\lambda}.$$

As consequence, by sending $\tau_1 \to 0$, one can recover the monotonicity of Perelman's reduced volume. *Vasyl Gorkaviy (Kharkov)*

Zbl 1196.53001
Brendle, Simon
Ricci flow and the sphere theorem.
Graduate Studies in Mathematics 111.
Providence, RI: American Mathematical Society (AMS)
(ISBN 978-0-8218-4938-5/hbk). vii, 176 p. \$ 47.00 (2010).

This book deals with some classical and recent results about R. Hamilton's Ricci flow. Special emphasis is put on convergence results, where one starts with a compact Riemannian manifold (M, g), assumes that g satisfies a certain condition (typically, positivity of some quantity coming from the curvature tensor), and shows that the Ricci flow $g(t)$ with initial condition $g(0) = g$ preserves the condition, is defined on some maximal time interval $[0, T)$, and that, as t tends to T, a suitable rescaling of $g(t)$ converges to some nice metric on M (e.g., a round metric), hereby establishing the existence of such a metric.

Historically, the first result of this form is Hamilton's foundational theorem on 3-manifolds of positive Ricci curvature. More recent important results include theorems by Böhm and Wilking on manifolds with positive curvature operator, and Brendle and Schoen on manifolds with positive isotropic curvature, which leads to a differentiable sphere theorem. This latter result is the main focus of the book under review.

Chapter 1 is a nice survey of Riemannian sphere theorems, i.e., theorems that give sufficient conditions for a Riemannian manifold to be homeomorphic (or sometimes even diffeomorphic) to the n-sphere. This chapter gives a good glimpse of various techniques that have been used to deal with such matters, some of them in nature rather different from those discussed in the rest of the book. This chapter contains not only statements but also sketches of proofs and key ideas, and provides an excellent introduction to the subject, as well as a motivation for the following chapters.

Chapters 2 and 3 are devoted to basic results on the Ricci flow. Chapter 4 deals with the Ricci flow on the 2-sphere and proves convergence to a round metric assuming that the initial metric has positive Gauss curvature. This result is due to Hamilton. The proof presented here includes elements from a paper by Chen, Lu and Tian.

In Chapter 5, more advanced results are discussed, in particular Hamilton's maximum principle for tensors. This is the main tool that allows to prove that some curvature conditions are preserved in higher dimensions. In Chapter 6, the material of Chapter 5 is applied to prove Hamilton's theorem on compact 3-manifolds with positive Ricci curvature, and the Hamilton-Ivey pinching estimate. The latter, while a digression from the main theme of the book, is a key ingredient in Perelman's proof of the geometrisation conjecture.

Chapters 7 and 8 tackle more recent material: several curvature conditions are shown to be preserved under Ricci flow, and corresponding convergence theorems are established. This chapter culminates with the proof of the Brendle-Schoen sphere theorem. Chapter 9 deals with further rigidity results, where typically positivity conditions are relaxed to nonnegativity. New ingredients are a 'strict' maximum principle and the notion of holonomy reduction.

Ricci flow is a very rich and flourishing theory, of which many aspects are of course not discussed in Brendle's book. Among important topics not covered, the reviewer would like to mention two: (1) *G. Perelman*'s work on the geometrisation of 3-manifolds [arXiv e-print service, Cornell University Library, Paper No. 0211159, electronic only (2002; Zbl 1130.53001); arXiv e-print service, Cornell University Library, Paper No. 0303109, electronic only (2003; Zbl 1130.53002); arXiv e-print service, Cornell University Library, Paper No.

0307245, electronic only (2003; Zbl 1130.53003)]; (see also [*B. Kleiner* and *J. Lott*, Geom. Topol. 12, No. 5, 2587–2855 (2008; Zbl 05530173); *J. Morgan* and *G. Tian*, Ricci flow and the Poincaré conjecture. Clay Mathematics Monographs 3. Providence, RI: American Mathematical Society (AMS); Cambridge, MA: Clay Mathematics Institute (2007; Zbl 1179.57045); *H.-D. Cao* and *X.-P. Zhu*, Asian J. Math. 10, No. 2, 165–492 (2006; Zbl 05071765); *L. Bessières, G. Besson, M. Boileau, S. Maillot* and *J. Porti*, Geometrisation of 3-manifolds, to appear in Tracts of the EMS (2010)]) and (2) the Kähler-Ricci flow (a good starting point is Chapter 2 of [*B. Chow* et al., The Ricci flow: techniques and applications. Part I: Geometric aspects. Mathematical Surveys and Monographs 135. Providence, RI: American Mathematical Society (AMS) (2007; Zbl 1157.53034)]). Other matters of current interest include stability results (which are convergence results, usually in infinite time, under conditions rather different from those considered by Brendle), Harnack inequalities, applications of Perelman's surgery technique for the Ricci flow (and its variants), existence-uniqueness theory for rough initial data, the Ricci flow on noncompact manifolds, and many others.

In conclusion, Brendle's book is an excellent introduction to the Ricci flow in the world of positive curvature. It is a welcomed addition to the other standard introductory texts on the subject, such as the books [*B. Chow* and *D. Knopf*, The Ricci flow: an introduction. Mathematical Surveys and Monographs 110. Providence, RI: American Mathematical Society (AMS) (2004; Zbl 1086.53085); *P. Topping*, Lectures on the Ricci flow. London Mathematical Society Lecture Note Series 325. Cambridge: Cambridge University Press (2006; Zbl 1105.58013); *B. Chow, P. Lu* and *L. Ni*, Hamilton's Ricci flow. Graduate Studies in Mathematics 77. Providence, RI: American Mathematical Society (AMS) (2006; Zbl 1118.53001)]. It can be read by graduate students and researchers with a general background in Riemannian geometry, and should be of interest for both beginners and experts of geometric flows.

Sylvain Maillot (Montpellier)

Breuillard, Emmanuel

Spellings: Breuillard, Emmanuel [15] Breuillard, E. [4]
Author-Id: breuillard.emmanuel
Publications: 19 including 18 Journal Article(s)

MSC 2010

11	**20** · Group theory and generalizations
6	**11** · Number theory
6	**22** · Topological groups, Lie groups
5	**60** · Probability theory and stochastic processes
3	**43** · Abstract harmonic analysis

more ...

Journals

3	Geometric and Functional Analysis - GAFA
2	Annals of Mathematics. Second Series
1	Electronic Research Announcements in Mathematical Sciences [electronic only]
1	Ergodic Theory and Dynamical Systems
1	Geometry & Topology

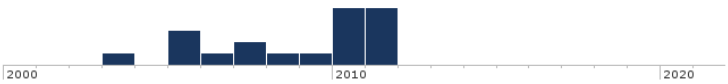
more ...

Co-Authors

5	Gelander, Tsachik
5	Green, Ben G.
3	Tao, Terence C.
1	de Cornulier, Yves
1	Friz, Peter K.

more ...

Publication Years



Zbl 05960722
Breuillard, Emmanuel
A height gap theorem for finite subsets of $\mathrm{GL}_d(\overline{\mathbb{Q}})$ and non-amenable subgroups.
Ann. Math. (2) 174, No. 2, 1057-1110 (2011).

The author introduces a conjugation invariant height $\hat{h}(F)$ on finite subsets of matrices F in $GL_d(\bar{\mathbb{Q}})$, where $\bar{\mathbb{Q}}$ is the field of algebraic numbers, which is well suited to the study of geometric and arithmetic behavior of power sets $F^n = F \cdots F$ for $n \in \bar{\mathbb{Q}}$. Among many interesting results of the paper the following two theorems are outstanding: Let F be a finite subset of $GL_d(\bar{\mathbb{Q}})$ generating a nonamenable subgroup that acts strongly irreducibly. Then there exists a constant $\epsilon(d) > 0$ such that $\hat{h}(F) > \epsilon(d)$. If F is a finite subset of $GL_d(\bar{\mathbb{Q}})$ generating a nonvirtually solvable subgroup, then there exists a constant $\epsilon(d) > 0$ such that $\hat{h}(F) > \epsilon(d)$. Using these two theorems the author can for instance prove the following statements: There is an $n(d) \in \mathbb{N}$ such that if K is a field and F is a finite symmetric subset of $GL_d(K)$ containing 1 which generates a nonvirtually solvable subgroup, then F^n contains two elements A and B which generate a non-abelian free subgroup. There is an an integer $n(d) \in \mathbb{N}$ such that if K is a field and F is finite subset of $GL_d(K)$

which generates an infinite subgroup, then $(F \cup F^{-1})^n$ contains an element of infinite order. The interpretation of $\hat{h}(F)$ in terms of spectral radius allows the author to derive the following two statements: There are constants $n(d) \in \mathbb{N}$ and $c(d) \in \mathbb{N}$ such that if F is any finite subset of $GL_d((\bar{\mathbb{Q}})$ containing 1, then there is some $A \in F^n$ and some eigenvalue λ of A such that $h(\lambda) \geq \frac{1}{|F^c|} \cdot \hat{h}(F)$, where $h(\lambda)$ is the height of λ in sense of [*E. Bompieri* and *W. Gubler*, Heights in Diophantic Geometry. New Mathematical Monographs 4. Cambridge: Cambridge University Press. (2006; Zbl 1115.11034)]. There are constants $n(d) \in \mathbb{N}$ and $\epsilon(d) > 0$ such that if F is any finite subset of $GL_d((\bar{\mathbb{Q}})$ containing 1 and generating a nonvirtually solvable subgroup, then one finds a matrix $A \in F^n$ and an eigenvalue λ of A such that $h(\lambda) > \epsilon(d)$. If F is is a finite subset of $GL_d((\bar{\mathbb{Q}})$, then $\hat{h}(F) = 0$ if and only if the group generated by F is virtually nilpotent. The author introduces for F also the minimal height and compares the minimal height $h(F)$ with the normalized height $\hat{h}(F)$. Using estimates relating the minimal norm of F defined by the author and the matrix coefficients of the elements of F in the adjoint representation he is able to obtain a global upper bound on the height of the matrix coefficients of the finite set F of matrices in $GL_d(\bar{\mathbb{Q}})$. The author gives in the paper many hints at the relations of his results to classical statements (e.g., Margulis Lemma, uniform Tits alternative, uniform version of the Burnside-Schur theorem) and to classical conjectures (e. g., the Lehmer problem).

Karl Strambach (Erlangen)

Zbl 06010060
Breuillard, Emmanuel; Green, Ben; Tao, Terence
Suzuki groups as expanders.
Groups Geom. Dyn. 5, No. 2, 281-299 (2011).

Let $\epsilon > 0$ be a real number. A graph X is called an ϵ-expander if, for all sets A consisting of at most half the vertices of X, we have $|\partial A| \geq \epsilon |A|$ where ∂A is the set of vertices of $X \setminus A$ that are joined to a vertex in A. The important particular case is a Cayley graph. Given a finite group G with a symmetric generating set S, one defines the Cayley graph $Cay(G, S)$ to be the graph with vertex set G in which vertices x and y are joined to an edge if and only if $x = ys$ for some $s \in S$. *M. Kassabov, A. Lubotzky* and *N. Nikolov* [Proc. Nat. Acad. Sci. USA 103, 6116–6119 (2006; Zbl 1161.20010)] announced the following result: There exist $k \in \mathbb{N}$ and $\epsilon > 0$ such that, for every non-abelian finite simple group G with the possible exception of the Suzuki groups $Sz(q)$, one may select a symmetric generating set S of k generators for which $Cay(G, S)$ is an ϵ-expander. The main aim of this paper is to remove the lacuna in this result. It is proved that there exists $\epsilon > 0$ such that, for every Suzuki group $G = Sz(q)$, one may select a generating pair a, b in G such that, setting $S := \{a^{\pm 1}, b^{\pm 1}\}$, the Cayley graph $Cay(G, S)$ is an ϵ-expander. In fact, the authors prove that a random pair of elements a, b will generate $Sz(q)$ and have the above expansion property with probability going to 1 as $q \to \infty$.

Anatoli Kondrat'ev (Ekaterinburg)

Figalli, Alessio

Spellings: Figalli, Alessio [41] Figalli, A. [9]
Author-Id: figalli.alessio
Publications: 50 including 1 Book(s) and 47 Journal Article(s)

MSC 2010

26	**49** · Calculus of variations and optimal control; optimization
22	**35** · Partial differential equations (PDE)
9	**53** · Differential geometry
8	**58** · Global analysis, analysis on manifolds
5	**37** · Dynamical systems and ergodic theory

more ...

Journals

4	Communications on Pure and Applied Mathematics
3	Calculus of Variations and Partial Differential Equations
3	European Series in Applied and Industrial Mathematics (ESAIM): Control, Optimization and Calculus of Variations
3	Journal of Functional Analysis
2	Archive for Rational Mechanics and Analysis

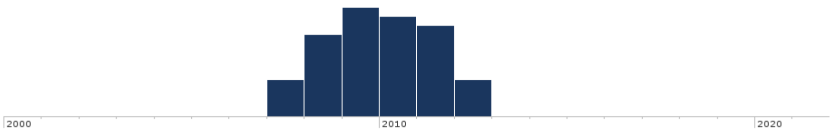
more ...

Co-Authors

7	Ambrosio, Luigi
7	Rifford, Ludovic
6	Villani, Cédric
3	Maggi, Francesco
3	Pratelli, Aldo M.

more ...

Publication Years



Zbl 1217.53038
Figalli, A.; Rifford, L.; Villani, C.
Tangent cut loci on surfaces.
Differ. Geom. Appl. 29, No. 2, 154-159 (2011).

Let (M, g) be a two-dimensional smooth compact Riemannian manifold, satisfying an appropriate convexity assumption on its tangent focal cut loci. The main result of the paper under review is that all injectivity domains of M are semiconvex. In particular, this implies that every tangent cut locus of M is an Alexandrov space, partially answering a question of *J.-I. Itoh* and *M. Tanaka* [Trans. Am. Math. Soc. 353, No. 1, 21–40 (2001; Zbl 971.53031)]. The convexity assumption mentioned above is that the (signed) curvature of the tangent focal loci of (M, g) at all focal cut velocities are uniformly bounded from below by a positive constant. This assumption is stable under small perturbations of the metric g in the C^4-topology. As a corollary, semiconvexity of injectivity domains is also preserved under such perturbations of g although the dependence of the cut locus on the metric is highly irregular. The proof of the main result is considerably short since it relies on previous results of the same authors [Calc. Var. Partial Differ. Equ. 39, No. 3–4, 307–332 (2010; Zbl 1203.53034) and "Nearly round spheres look convex", Am. J. Math., to appear]. *Renato G. Bettiol (Notre Dame)*

1195.46032
Figalli, Alessio
On flows of $H^{3/2}$-vector fields on the circle.
Math. Ann. 347, No. 1, 43-57 (2010).

The universal Teichmüller space $T(1)$ can be regarded as a group formed by quasi-symmetric maps on the unit circle $\mathbb{S}^1$. It is naturally endowed with a well-known complex Hilbert manifold structure that is compatible with the Weil-Petersson metric, making the identity component a topological group. The aim of the paper under review is to characterize the flows of the vector fields defined by the tangents to the identity components (consisting of $H^{3/2}$-vector fields on $\mathbb{S}^1$, see the definition below) in terms of fractional Sobolev norms.

Denote by $W^{3/2,2}(\mathbb{S}^1)$ the subspace of $L^2(\mathbb{S}^1)$ consisting of functions whose weak derivatives are in $W^{1/2,2}(\mathbb{S}^1)$, where we introduce the $W^{s,p}(\mathbb{S}^1)$-norm for $s \in (0,1)$ and $p \in [1,\infty)$ as follows:

$$\|v\|_{W^{s,p}(\mathbb{S}^1)} = \|v\|_{L^p(\mathbb{S}^1)} + \left(\int_{\mathbb{S}^1}\int_{\mathbb{S}^1} \frac{|v(x)-v(y)|^p}{|x-y|^{1+sp}}\,dx\,dy\right)^{1/p}.$$

Since $\mathbb{S}^1$ is compact, it is well-known that $W^{3/2,2}(\mathbb{S}^1)$ is identified with the Sobolev space $H^{3/2}(\mathbb{S}^1)$ [see *R. A. Adams*, "Sobolev spaces" (Pure and Applied Mathematics 65; New York–San Francisco–London: Academic Press) (1975; Zbl 0314.46030)] for the definition and basic properties of the space $H^{3/2}(\mathbb{S}^1)$).

Let T be a positive number. For $u \in C([0,T], H^{3/2}(\mathbb{S}^1))$, we consider the following ordinary differential equation:

$$\begin{cases} \dot{f}(t,x) &= u(t, f(t,x)), \quad x \in \mathbb{S}^1, \\ f(0,x) &= x. \end{cases} \tag{A}$$

The author proves that $H^{3/2}$ embeds into logLipschitz and thus there exists a unique solution $f(t,x)$ for (A) that is a self-homeomorphism of $\mathbb{S}^1$ for all $t \in [0,T]$ [see, for instance, *P. Hartman*, "Ordinary differential equations" (2nd ed., Reprint; Boston–Basel–Stuttgart: Birkhäuser) (1982; Zbl 0476.34002)]. More precise information is given in the main result of this paper:

Given a vector field $u \in C([0,T], H^{3/2}(\mathbb{S}^1))$, let $f(t,\cdot) : \mathbb{S}^1 \to \mathbb{S}^1$ be the corresponding flow map. Then $f(t,\cdot)$ belongs to $W^{1,p}(\mathbb{S}^1)$ for all $p \in [1,\infty)$ and to $W^{1+r,q}(\mathbb{S}^1)$ for all $r \in (0,1/2)$ and $q \in [1,1/r)$. On the other hand, there exists an autonomous vector field $u \in H^{3/2}(\mathbb{S}^1)$ such that its flow map is neither Lipschitz nor $W^{1+r,1/r}$ for all $r \in (0,1)$. In particular, the flow map is not $H^{3/2}$.

The first part of the theorem is settled by Proposition 2.1. To prove the remaining part of the theorem, the author shows that

$$u_x \circ f(t,\cdot) \in L^\infty([0,T], W^{r,q}(\mathbb{S}^1))$$

for $r \in (0,1/2)$ and $q \in [1,1/r)$ (Lemma 2.2). Then the author is able to estimate

$$\tfrac{d}{dt}\left|f_x(t,x)-f_x(t,y)\right| \le \left|u_x(t,f(t,x))-u_x(t,f(t,y))\right|\left|f_x(t,x)\right| + \\ \left|u_x(t,f(t,y))\right|\left|f_x(t,x)-f_x(t,y)\right|.$$

Duhamel's formula along with the initial condition $f_x(0,x)=1$ yields that

$$\left|f_x(t,x)-f_y(t,y)\right| \le \\ \int_0^t e^{\int_s^t \left|u_x(\tau,f(\tau,y))\right| d\tau} \left|u_x(s,f(s,x))\right| - u_x(s,f(s,y)) \left|f_x(s,x)\right| ds.$$

Using Hölder's and Jensen's inequalities, the author obtains

$$\frac{\left|f_x(t,x)-f_x(t,y)\right|^q}{\left|x-y\right|^{1+rq}} \in L^1(\mathbb{S}^1\times\mathbb{S}^1)$$

uniformly in time. It follows that

$$f_x \in L^\infty\left([0,T], W^{r,q}(\mathbb{S}^1)\right) \text{ for all } r\in\left(0,\frac{1}{2}\right) \text{ and } q\in\left[1,\frac{1}{r}\right).$$

In the last part of the paper, the author constructs a counterexample to the $H^{3/2}$-regularity. *Chaohui Zhang (Atlanta)*

Ioana, Adrian

Spellings: Ioana, Adrian [13]
Author-Id: ioana.adrian
Publications: 13 including 13 Journal Article(s)

MSC 2010

9	**37** · Dynamical systems and ergodic theory
9	**46** · Functional analysis
3	**28** · Measure and integration
1	**03** · Mathematical logic
1	**20** · Group theory and generalizations

more ...

Journals

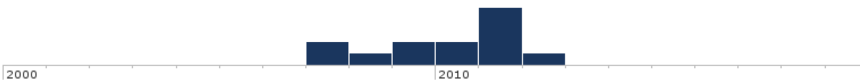
2	Journal of Functional Analysis
1	Acta Mathematica
1	Advances in Mathematics
1	Annales Scientifiques de l'École Normale Supérieure. Quatrième Série
1	Duke Mathematical Journal

more ...

Co-Authors

3	Chifan, Ionuţ
1	Kechris, Alexander S.
1	Peterson, Jesse D.
1	Popa, Sorin Teodor
1	Tsankov, Todor

Publication Years



Zbl 1230.37010
Ioana, Adrian
Orbit inequivalent actions for groups containing a copy of $\mathbb{F}_2$.
Invent. Math. 185, No. 1, 55-73 (2011).

Free, ergodic, probability measure preserving actions of countable discrete groups Γ_1, Γ_2 on standard probability spaces X_1, X_2 are said to be orbit equivalent if there is an isomorphism $\theta : X_1 \to X_2$ of measure spaces with $\theta(\Gamma_1 x) = \Gamma_2(\theta x)$ almost everywhere (that is, if the two actions induce measurably isomorphic equivalence relations). *H. A. Dye* [Am. J. Math. 81, 119–159 (1959; Zbl 87.11501)] showed that all such actions are orbit equivalent if Γ_1 and Γ_1 are abelian, and this was extended to all amenable groups in work of *D. S. Ornstein* and *B. Weiss* [Bull. Am. Math. Soc., New Ser. 2, 161–164 (1980; Zbl 427.28018)] and *A. Connes*, *J. Feldman* and *B. Weiss* [Ergodic Theory Dyn. Syst. 1, 431–450 (1981; Zbl 491.28018)]. For non-amenable groups the situation is more rigid, and *A. Connes* and *B. Weiss* [Isr. J. Math. 37, 209–210 (1980; Zbl 479.28017)] (and others) showed that a non-amenable group admits at least two actions that are not orbit equivalent. More recently, it has been shown that groups with property (T), free groups, weakly rigid groups, non-amenable products of infinite groups and mapping class groups all have the property that they admit uncountably many actions that are not orbit equivalent. In this paper another large class of groups is added to this list, with a proof that the same property holds for any group containing a copy of the free group on two generators. This result covers most – but not all – non-amenable groups. The proof realizes the free group as a subgroup of finite index in $SL_2(\mathbb{Z})$, and then exploits the relative property (T).

Thomas B. Ward (Norwich)

Zbl 1235.37005
Ioana, Adrian
Cocycle superrigidity for profinite actions of property (T) groups.
Duke Math. J. 157, No. 2, 337-367 (2011).

The main result of the paper is Theorem A on orbit equivalence (OE) superrigidity. As a consequence of this result, the author constructs uncountably many non-OE profinite actions for the arithmetic groups $SL_n(\mathbb{Z})$ ($n \geq 3$) and their finite subgroups, as well as for the groups $SL_m(\mathbb{Z}) \ltimes \mathbb{Z}^m$ ($m \geq 2$). The author deduces Theorem A as a consequence of a theorem (Theorem B) on cocycle superrigidity.

"Let $\Gamma \curvearrowright X$ be a free ergodic measure-preserving profinite action (i.e., an inverse limit of actions $\Gamma \curvearrowright X_n$ with X_n finite) of a countable property (T) group Γ (more generally, of a group Γ which admits an infinite normal subgroup Γ_0 such that the inclusion $\Gamma_0 \subset \Gamma$ has relative property (T) and Γ/Γ_0 is finitely generated) on a standard probability space X. The author proves that if $\omega : \Gamma \times X \to \Lambda$ is a measurable cocycle with values in a countable group Λ, then ω is a cohomologous to a cocycle ω' which factors through the map $\Gamma \times X \to \Gamma \times X_n$, for some n. As a corollary, he shows that any free ergodic measure-preserving action $\Lambda \curvearrowright Y$ comes from a (virtual) conjugacy of actions."

The notion of property (T) for locally compact groups was defined by *D. A. Kazhdan* [Funct. Anal. Appl. 1, 63–65 (1967); translation from Funkts. Anal. Prilozh. 1, No. 1, 71–74 (1967; Zbl 168.27602)] and the notion of relative property (T) for inclusion of countable groups $\Gamma_0 \subset \Gamma$ was defined by *G. A. Margulis* [Ergodic Theory Dyn. Syst. 2, 383–396 (1982; Zbl 532.28012)].

The concept of superrigidity was introduced by *G. D. Mostow* [Strong rigidity of locally symmetric spaces. Annals of Mathematics Studies. No. 78. Princeton, NJ: Princeton University Press and University of Tokyo Press (1973; Zbl 265.53039)] and by *G. A. Margulis* [Discrete subgroups of semisimple Lie groups. Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, 17. Berlin: Springer (1991; Zbl 0732.22008)] in the context of studying the structure of lattices in rank one and higher rank Lie groups, respectively.

The paper under review presents a new class of orbit equivalent superrigid actions. Previously, the first result of this type was obtained by *A. Furman* [Ann. Math. (2) 150, No. 3, 1059–1081 (1999; Zbl 0943.22013); ibid. 150, No. 3, 1083–1108 (1999; Zbl 943.22012)], who combined the cocycle superrigidity by *R. J. Zimmer* [Ergodic theory and semisimple groups. Mono-

graphs in Mathematics, Vol. 81. Boston-Basel-Stuttgart: Birkhäuser (1984; Zbl 571.58015)] with ideas from geometric group theory to show that the actions $SL_n(\mathbb{Z}) \curvearrowright T^n (n \geq 3)$ are OE superrigid. The deformable actions of rigid groups are OE superrigid by [*S. Popa*, in: Proceedings of the international congress of mathematicians (ICM) 2006. Volume I: Plenary lectures and ceremonies. Zürich: European Mathematical Society (EMS). 445–477 (2007; Zbl 1132.46038)].
A detailed analysis of several applications illustrates very well the most important points of the author's approach. *Nikolaj M. Glazunov (Kyïv)*

Lewin, Mathieu

Spellings: Lewin, Mathieu [31] Lewin, M. [2]
Author-Id: lewin.mathieu
Publications: 32 including 30 Journal Article(s)

MSC 2010

28	**81** · Quantum Theory
8	**35** · Partial differential equations (PDE)
7	**82** · Statistical mechanics, structure of matter
5	**49** · Calculus of variations and optimal control; optimization
4	**47** · Operator theory

more ...

Journals

4	Communications in Mathematical Physics
3	Annales Henri Poincaré
3	Archive for Rational Mechanics and Analysis
2	Advances in Mathematics
2	Nonlinearity

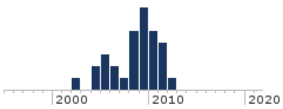
more ...

Co-Authors

10	Hainzl, Christian
7	Séré, Eric
4	Solovej, Jan Philip
3	Cancès, Eric
3	Lenzmann, Enno

more ...

Publication Years



Zbl 1180.81155
Gravejat, Philippe; Lewin, Mathieu; Séré, Éric
Ground state and charge renormalization in a nonlinear model of relativistic atoms.
Commun. Math. Phys. 286, No. 1, 179-215 (2009).

$D^0 := -i\sum_{k=1\sim 3}\alpha_k\partial_k + \beta$, $\alpha_j, \beta : 4\times 4$ Dirac matrices. P: a self-adjoint operator $0 \le P \le 1$ acting on $H = L^2(\mathbb{R}^3, \mathbb{C}^4)$. $P^0_- := \chi_{(-\infty,0]}(D^0)$ is the negative spectral projector of D^0. $P^0_+ := 1 - P^0_-$, $Q = P - P^0_-$. $D^\zeta(p) := (\alpha p + \beta)(1 + \zeta(|p|^2/\Lambda^2))$, $\zeta : [0,\infty) \to [0,\infty)$, Λ ultraviolet cut-off. The authors study the rBDF energy which descrives the relativistic electrons interacting with the Dirac sea, in an external electrostatic potential:

$$\varepsilon^\nu_r(Q) :=$$
$$\operatorname{tr}(P^0 - D^\zeta Q P^0_-) + \operatorname{tr}(P^0_+ D^\zeta Q P^0_+) - \alpha D(\nu D(\nu(x), \rho_Q(x)) + (\alpha/2) D(\rho_Q(x), \rho_Q(x)),$$

$$D(f,g) = 4\pi \int_{\mathbb{R}^3} |k|^{-2}\overline{\widehat{f}(k)}\widehat{g}(k)\,dk.$$

$\rho_Q(x) = \mathrm{tr}_{\mathbb{C}^4}(Q(x,x))$ for Q acting on H with kernel $Q(x,y)$.
Minimizers: $E_r^\nu(q) = \underset{Q\in K(q)}{} \to \inf \varepsilon_r^\nu(Q)$ in $K(q) = \{Q; -P_-^0 \le Q \le P_+^0\}$,

$$\{\mathrm{tr}(P_-^0 Q P_-^0) + \mathrm{tr}(P_+^0 Q P_+^0) = q\} \subset \{Q; \mathrm{tr}(|Q|^2) < \infty\}.$$

Results:

(I) The minimizers exist if and only if $q \in [q_m, q_M]$.

(II) In a nonrelativistic limit "$\alpha \to 0$, $\Lambda \to \infty$ such that $\alpha \log \Lambda \to 9$", one obtains $0 = q_m < Z \le q_M \le 2Z$, an estimate on the number of electrons bounded by a nucleus of charge $Z = \int \nu(x)\,dx$.

(III) $Q = \chi_{(-\infty,\mu)}(D_Q) - P_-^0 + \delta$, $D_Q = D^0 + \alpha(\rho_Q - \nu)\ddagger|\cdot|^{-1}$.

(IV) $\rho_Q(x) \in L^1(\mathbb{R}^3)$, and $\int_{\mathbb{R}^3} \rho_Q(x)\,dx - Z = (q-Z)/(1+\alpha B_\Lambda^\zeta(0))$, $\alpha_{\text{phys}} \sim \alpha(1 + 2\alpha/(3\pi/(3\pi)\log\Lambda)^{-1}$ hold.

Hideo Yamagata (Osaka)

Zbl 1216.81180
Lewin, Mathieu
Geometric methods for nonlinear many-body quantum systems.
J. Funct. Anal. 260, No. 12, 3535-3595 (2011).

The author presents a geometric formalism able to handle particular nonlinear problems appearing in quantum mechanics of many particle systems, specially when some of these particles are allowed to escape at infinity. For that purpose a suitable weak topology is introduced leading to the notion of geometric convergence involving state spaces with variable dimensions. As an application, an elegant proof of the HVZ (Hunziker-Van Winter-Zhislin) theorem is provided. Links of this geometric convergence notion with the geometrical localization method introduced by Dereziński and Gerard are then clarified. Several applications are finally provided: the finite-rank approximation (known as Multi Configuration Approximation in quantum chemistry), the analysis of some translation-invariant systems involving nonlinear effective interactions (appearing in low energy Nuclear Physics) and the multi-polaron system, an effective model in electron-phonon interaction.

Manolescu, Ciprian

Spellings: Manolescu, Ciprian [10]
Author-Id: manolescu.ciprian
Publications: 10 including 9 Journal Article(s)

MSC 2010

10	**57** · Manifolds and cell complexes
2	**53** · Differential geometry
1	**55** · Algebraic topology

Journals

2	Duke Mathematical Journal
2	Geometry & Topology
1	Advances in Mathematics
1	Annals of Mathematics. Second Series
1	IMRN. International Mathematics Research Notices

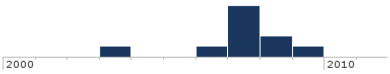
more ...

Co-Authors

3	Ozsváth, Peter S.
1	Lipshitz, Robert
1	Owens, Brendan
1	Sarkar, Sucharit
1	Szabó, Zoltán Imre

more ...

Publication Years



Zbl 1195.57032
Manolescu, Ciprian; Ozsváth, Peter
On the Khovanov and knot Floer homologies of quasi-alternating links.
Akbulut, Selman (ed.) et al., Proceedings of the 14th Gökova geometry-topology conference, Gökova, Turkey, May 28–June 2, 2007. Cambridge, MA: International Press (ISBN 978-1-57146-107-0/pbk). 60-81 (2008).

Khovanov homology and knot Floer homology are two invariants of links which have been extensively studied in the recent years. Those invariants take the form of bi-graded R-modules, where in the paper under review R is either $\mathbb{Z}$ or $\mathbb{Z}/2\mathbb{Z}$. If $\{i, j\}$ denote the bi-grading of any of the two theories we set the δ-grading to be $\delta = j - i$.
Extensive computations of Bar Natan in *D. Bar-Natan* [Algebr. Geom. Topol. 2, 337–370 (2002; Zbl 0998.57016)] show that the majority of prime knots with small crossing number have Khovanov homology concentrated in one particular δ-grading: $-\sigma/2$, where σ is the signature of the knot. The authors refers to such links as Khovanov homologically σ-thin. Floer homologically σ-thin links are defined in a similar way for knot Floer homology (this definition coincides with the definition of perfect knots in *J. Rasmussen* [Duke Math. J. 136, No. 3, 551–583 (2007; Zbl 1125.57004)]).
In [The support of the Khovanov's invariants for alternating knots, preprint 2002, math/0201105], *Eun Soo Lee* shows that alternating links are Khovanov

σ-homologically thin. Since 197 out of the 250 knots up to 10 crossing are alternating this gives a partial explanation to Bar Natan's observation.
On the Heegard Floer side, it was shown by Rasmussen in *J. Rasmussen* [loc. cit.] that 2-bridge knots are Floer homologically σ-thin, this was then generalized by *Z. Szabó* and *P. Ozváth* in [Geom. Topol. 7, 225–254 (2003; Zbl 1130.57303)] to alternating knots.
The paper under review generalizes those results to the class of quasi-alternating links. Precisely the main results of the paper are:
Theorem 1.1. Quasi-alternating links are Khovanov homologically σ-thin over $\mathbb{Z}$.
Theorem 1.2. Quasi-alternating links are Floer homologically σ-thin over $\mathbb{Z}/2\mathbb{Z}$.
Quasi-alternating links were defined in *P. Ozsváth* and *Z. Szabó* [Adv. Math. 194, No. 1, 1–33 (2005; Zbl 1076.57013)] where it was also shown that alternating links are quasi-alternating.
83 of the 85 prime knots of crossing number less than nine are Khovanov and Floer homologically σ-thin. 82 of those are known to be quasi-alternating. Thus, with the exception of the 9_{46} knot (for which it is not known whether it is quasi-alternating or not), this gives a complete explanation of the predominance of homologically σ-thin knots. Consequences on the Heegard Floer homology of Dehn surgeries on quasi-alternating knots are also discussed.
The proof of both theorems relies on the skein exact triangle relating the homology of L to the homology of the resolutions L_0 and L_1 of some crossing of a planar projection of L. Note that the same resolutions are used to recursively define quasi-alternating links in *P. Ozsváth* and *Z. Szabó* [loc.cit.]. For the Khovanov case this exact triangle is a consequence of its definition and for the knot Floer case in was proved in *C. Manolescu* [Math. Res. Lett. 14, No. 5–6, 839–852 (2007; Zbl 1161.57005)]. In the first case, the behavior of the grading in the triangle was described in *M. Khovanov* [Duke Math. J. 101, No. 3, 359–426 (2000; Zbl 0960.57005)]. However in the second case, it was not known whether the triangle leads to δ-graded long exact sequence. The paper under review carefully studies the behavior of the δ-grading and its relation with the signature in both cases to conclude the main results.
In Section 2 the Khovanov case is treated where it is shown that the maps appearing in the triangle preserve the grading up to a shift determined by the signature of the quasi-alternating link; this allows to prove Theorem 1.1 using the recursive definition of quasi-alternating links.
The more involved case of knot Floer homology is treated in Section 3 where it is shown that two of the three maps in the triangle preserve the δ-grading up to the same shift of Section 2. A last remark concerning the relation between the determinants of the quasi-alternating link and its resolutions allows to show that the third map is 0 in this case; this allows to conclude Theorem 1.2 using the same argument as in Section 2. *Baptiste Chantraine (Bruxelles)*

1179.57022
Manolescu, Ciprian; Ozsváth, Peter S.;
Sarkar, Sucharit
A combinatorial description of knot Floer homology.
Ann. Math. (2) 169, No. 2, 633-660 (2009).

Knot Floer homology is a powerful knot invariant defined by Ozsváth and Szabó, and generalizes the Alexander-Conway polynomial. It was originally defined using a filtered chain complex whose differential counted pseudo-holomorphic disks, and hence it was not possible to compute it algorithmically.

This fundamental paper presents an algorithm for computing both the hat and the minus versions of knot Floer homology of a given knot K. It uses a grid presentation of K, from which one can construct a multi-pointed Heegaard diagram for K. The Heegaard surface is a torus, and the α curves are all meridians and the β curves are all longitudes. If the number of α curves is n, then there are exactly $2n$ marked points that specify K. Every pseudo-holomorphic disk can be shown to correspond to a rectangle in this grid. The authors give combinatorial formulas for the Alexander and Maslov gradings of the generators of the knot Floer chain complex, and the differential can be computed by counting empty rectangles in the grid. Even though this method is algorithmic, it is by no means efficient.

The concordance invariant τ and link Floer homology can also be computed algorithmically from this picture. It is worth noting that this is a continuation of the work of Sarkar and Wang, who gave an algorithm for computing the hat version of Heegaard Floer homology of closed 3-manifolds.

A. Juhasz (Cambridge)

Miermont, Grégory

Spellings: Miermont, Grégory [21] Miermont, Grégory [2]
Author-Id: miermont.gregory
Publications: 23 including 1 Book(s) and 17 Journal Article(s)

MSC 2010

22	**60** · Probability theory and stochastic processes
7	**05** · Combinatorics
2	**92** · Applications of mathematics to Biology and other natural sciences
1	**00** · General mathematics
1	**11** · Number theory

more ...

Journals

4	Electronic Journal of Probability [electronic only]
4	Probability Theory and Related Fields
3	The Annals of Probability
2	Random Structures & Algorithms
1	Annales Scientifiques de l'École Normale Supérieure. Quatrième Série

more ...

Co-Authors

4	Haas, Bénédicte
4	Pitman, Jim W.
3	Aldous, David J.
3	Bertoin, Jean
2	Berestycki, Julien

more ...

Publication Years

Zbl 1102.60006
Bertoin, Jean; Miermont, Grégory
Asymptotics in Knuth's parking problem for caravans.
Random Struct. Algorithms 29, No. 1, 38-55 (2006).

A generalized version of Knuth's parking problem is considered, in which instead of cars drops of paint are distributed at random on the unit circle. More precisely, let $s_1, s_2, \dots, s_m$ be m locations on the unit circle (selected according to some probabilistic procedure described in the paper) and let $p_1, \dots, p_m$ be the (random) sizes of m drops of paint of total mass 1. These drops fall successively at the respective locations. Each time a drop of paint falls we brush it clockwise in such a way that the resulting painted portion of the circle is covered by a unit density of paint (i.e., no piece of circle is brushed twice). In this setting drops of paint may be viewed as a continuous version of m car caravans of total size n arriving at random on a circle with n parking spots.
Extending a recent paper of *P. Chassaing* and *G. Louchard* [Random Struct. Algorithms 21, No. 1, 76–119 (2002; Zbl 1032.60003)] the authors relate the asymptotics of the sizes of blocks formed by the painted pieces of the circle with the dynamics of the additive coalescence described by *D. J. Aldous* and *J. Pitman* [Ann. Probab. 26, No. 4, 1703–1726 (1998; Zbl 0936.60064)] and, imposing different assumptions on the tail distribution of the drops' size,

characterize several qualitatively different versions of the eternal additive coalescence.

Zbl 1071.60065
Miermont, Grégory
Self-similar fragmentations derived from the stable tree. II: Splitting at nodes.
Probab. Theory Relat. Fields 131, No. 3, 341-375 (2005).

[For part I see ibid. 127, No. 3, 423–454 (2003; Zbl 1042.60043).]
The author investigates a fragmentation process F^+ derived by a splitting procedure at branchpoints of the so-called stable tree $\mathcal{T}$ introduced by *T. Duquesne* and *J.-F. Le Gall* ["Random trees, Lévy processes and spatial branching processes" (2002; Zbl 1037.60074)]. The work is mainly motivated by the aim to generalise existing work on fragmentation processes on Brownian continuum random trees studied by *D. Aldous* and *J. Pitman* [Ann. Probab. 26, No. 4, 1703–1726 (1998; Zbl 0936.60064)] and *J. Bertoin* [Probab. Theory Relat. Fields 121, No. 3, 301–318 (2001; Zbl 0992.60076) and Ann. Inst. Henri Poincaré, Probab. Stat. 38, No. 3, 319–340 (2002; Zbl 1002.60072)].
Given a one-dimensional α-stable process (X_t) for $\alpha \in (1,2)$ with positive jumps only a height process (H_t) is constructed via a local time of the reflected processes of (X_t). Then (H_t) is used to define the tree $\mathcal{T}$ by equivalence classes on $[0,1]$. Furthermore, restricting the Lebesgue measure on $[0,1]$ on $\mathcal{T}$ one establishes a mass measure μ on $\mathcal{T}$. To each branchpoint $b \in \mathcal{T}$ a standard exponential random variable e_b is associated used to define on $\mathcal{T}$ an equivalence relation $\sim_t$ by $v \sim_t w$ if the path from v to w does not contain a branchpoint b with $e_b < tL(b)$ where $L(b)$ is a local time or width of the branchpoint b. The resulting equivalence classes are again trees and sorting these in decreasing order according to their masses $\mu(\mathcal{T}_j^t)$ the fragmentation process $F^+(t) = (\mu(\mathcal{T}_1^t), \mu(\mathcal{T}_2^t), \dots)$ is obtained. The author shows that F^+ is a self-similar fragmentation process with index $1/\alpha$ and erosion coefficient 0; the dislocation measure is characterised in terms of a stable subordinator with index $1/\alpha$ and its sequence of jumps. Moreover, explicit formulae for the semigroup as well as small/large time asymptotics are provided. Eventually, an alternative construction of F^+ is presented using path properties of (X_t).

Eckhard Giere (Clausthal-Zellerfeld)

Morel, Sophie

Spellings: Morel, Sophie [5]
Author-Id: morel.sophie
Publications: 5 including 1 Book(s) and 3 Journal Article(s)

MSC 2010

5	**14** · Algebraic geometry
4	**11** · Number theory
2	**20** · Group theory and generalizations
1	**05** · Combinatorics

Journals

1	Compositio Mathematica
1	Journal of the American Mathematical Society
1	Mathematische Zeitschrift

Publication Years

1233.11069
Morel, Sophie
On the cohomology of certain noncompact Shimura varieties.
With an appendix by Robert Kottwitz.
Annals of Mathematics Studies 173.
Princeton, NJ: Princeton University Press (ISBN 978-0-691-14293-7/pbk; 978-0-691-14292-0/hbk; 978-1-400-83539-3/ebook). xi, 217 p.
\$ 39.50, £ 27.95/pbk; \$ 75.00, £ 52.00/hbk; \$ 39.50/ebook (2010).

From the preface: "The goal of this text is to calculate the trace of a Hecke correspondence composed with a (big enough) power of the Frobenius automorphism at a good place on the intersection cohomology of the Baily-Borel compactification of certain Shimura varieties, and then to stabilize the result for the Shimura varieties associated to unitary groups over $\mathbb{Q}$.
"The main result is theorem 8.4.3. It expresses the above trace in terms of the twisted trace formula on products of general linear groups, for well-chosen test functions."
A very brief summary: Certain facts about the fixed point formula are reviewed and generalized in chapter 1. In chapters 2–6, the author treats the stabilization of the fixed point formula. After giving applications of this last in chapters 7 and 8, she proves a case of the twisted fundamental lemma in chapter 9. In the appendix to the monograph, R. Kottwitz compares two versions of twisted transfer factors. *Junecue Suh (Cambridge)*

Sanders, Tom

Spellings: Sanders, Tom [19]
Author-Id: sanders.tom
Publications: 19 including 19 Journal Article(s)

MSC 2010

18	**11** · Number theory
8	**43** · Abstract harmonic analysis
3	**20** · Group theory and generalizations
2	**22** · Topological groups, Lie groups
2	**42** · Fourier analysis

more ...

Journals

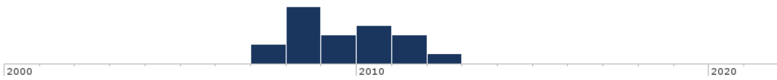
2	Acta Arithmetica
2	Annals of Mathematics. Second Series
2	Geometric and Functional Analysis - GAFA
2	Journal d'Analyse Mathématique
2	Mathematical Proceedings of the Cambridge Philosophical Society

more ...

Co-Authors

2	Green, Benjamin
1	Ruzsa, Imre Z.

Publication Years



Zbl 1213.43007
Sanders, Tom
A quantitative version of the non-Abelian idempotent theorem.
Geom. Funct. Anal. 21, No. 1, 141-221 (2011).

Let G be a finite (not necessarily abelian) group with Haar measure μ. Any complex-valued function f on G induces a convolution operator L_f from $L^2(G)$ to $L^2(G)$ by $g \mapsto f * g$, where

$$f * g(x) := \int f(y)g(y^{-1}x)\,d\mu(y).$$

It is not difficult to see that the algebra norm defined by

$$\|f\|_{A(G)} := \sup\{|\langle f, g\rangle_{L^2(\mu)} : \|L_g\| \leq 1\},$$

where $\|\cdot\|$ denotes the operator norm, reduces to the well known Wiener algebra in the case when G is abelian, i.e. $\|f\|_{A(G)} = \|\hat{f}\|_{\ell^1}$.
It is also relatively straightforward to verify, even in the non-abelian case, that the indicator function of a coset of a subgroup has $A(G)$ norm equal to 1. The triangle inequality implies therefore that small plus/minus combinations of such indicator functions also have small $A(G)$ norm.
The paper under review establishes the following converse of this observation, stating that all functions with small $A(G)$ norm are essentially of this form.

Theorem. Let $A \subseteq G$ be such that $\|1_A\|_{A(G)} \leq M$. Then there is an integer $L = L(M)$, subgroups $H_1, \ldots, H_L \leqslant G$, elements $x_1, \ldots, x_L \in G$ and signs $\sigma_1, \ldots, \sigma_L \in \{-1, 0, 1\}$ such that

$$1_A = \sum_{i=1}^{L} \sigma_i 1_{x_i H_i},$$

where L may be taken to be at most triply tower in $O(M)$.
Inspired by a result of *P. Cohen* [Am. J. Math. 82, 191–212 (1960; Zbl 0099.25504)], the abelian case of this theorem had been proved by *B. Green* and *T. Sanders* in [Ann. Math. (2) 168, No. 3, 1025–1054 (2008; Zbl 1170.43003)], and a qualitative version of the non-abelian theorem is due to *M. Lefranc* [C. R. Acad. Sci., Paris, Sér. A 274, 1882–1883 (1972; Zbl 0247.43014)].
Proving the theorem above, however, is not the only aim of the paper under review. Perhaps even more importantly, the author develops a version of the non-abelian Fourier transform relative to a suitable notion of "approximate group", widely used in the abelian context since the work of *J. Bourgain* [Geom. Funct. Anal. 9, No. 5, 968–984 (1999; Zbl 0959.11004)]. This allows him to transfer several results from additive combinatorics, including a Freiman-type result, to the non-abelian setting.
Although the paper contains an excellent introduction to guide the reader, he or she is well advised, given its conceptual as well as methodological complexity, to be comfortable with the general strategy and technical tools of [Green and Sanders, loc. cit.] before embarking on the present paper.

Julia Wolf (Palaiseau)

1209.43003
Sanders, Tom
Chowla's cosine problem.
Isr. J. Math. 179, 1-28 (2010).

Let G be an abelian group, to be thought of as discrete. For a finite symmetric subset $A \subseteq G$, one can ask how large the negative Fourier coefficients of the indicator function 1_A can be. (Note that the largest positive Fourier coefficient is trivially equal to the size of the set A. Also, since the set A is symmetric, the Fourier coefficients of 1_A are real, thus the question regarding the maximal negative value of the Fourier transform is well defined.)
We shall give a very brief history of the problem before stating the results of this paper. For a set $A \subseteq G$ as above, define

$$M_G(A) = \sup_{\gamma \in \widehat{G}} -\widehat{1_A}(\gamma).$$

S. Chowla [J. Reine Angew. Math. 217, 128–132 (1965; Zbl 0127.02104)] asked for a lower bound on $M_{\mathbb{Z}}(A)$. A simple averaging argument and the Littlewood conjecture [*S. V. Konyagin*, Izv. Akad. Nauk SSSR, Ser. Mat. 45, 243–265

(1981; Zbl 0493.42004); *O. C. McGehee, L. Pigno* and *B. Smith*, Ann. Math. (2) 113, 613–618 (1981; Zbl 0473.42001)] imply that $M_{\mathbb{Z}}(A) = \Omega(\log |A|)$. The best known bound is due to *I. Z. Ruzsa* [Acta Arith. 111, No. 2, 179–186 (2004; Zbl 1154.11312)] and of the form $M_{\mathbb{Z}}(A) = \exp(\Omega(\sqrt{\log |A|}))$.
Littlewood's conjecture has recently been extended to abelian groups other than $\mathbb{Z}$ by *B. Green* and *S. Konyagin* [Can. J. Math. 61, No. 1, 141-164 (2009; Zbl 05549875)]. Their results imply, for example, that $M_{\mathbb{Z}/p\mathbb{Z}}(A) = \log^{\Omega(1)} |A|$ for p a prime, provided that $|A| = (p+1)/2$.
In the current paper the author is able to improve on this and obtain the bound $M_{\mathbb{Z}/p\mathbb{Z}}(A) = \Omega(p^{1/3})$, again provided that $|A| = (p+1)/2$. For comparison, *J. Spencer* showed in [Trans. Am. Math. Soc. 289, 679–706 (1985; Zbl 0577.05018)] that there exist sets $A \subseteq \mathbb{Z}/p\mathbb{Z}$ of size $(p+1)/2$ such that $M_{\mathbb{Z}/p\mathbb{Z}}(A) = O(p^{1/2})$.
In more general abelian groups there is a simple but devastating obstacle to the obvious extension of the above result: if H is a finite subgroup of G, then $M_G(H) = 0$. The author therefore proves the following refinement, which is easily seen to imply the statement for $\mathbb{Z}/p\mathbb{Z}$ above.
Theorem. Suppose that G is a finite abelian group and A a symmetric subset of G with $|A| = \Omega(|G|)$. Then there is a subgroup $H \leq G$ such that

$$M_G(A) = |A \Delta H|^{\Omega(1)}.$$

The example of a set A consisting of a large finite subgroup together with a handful of other points shows that this result is best possible up to a power.
Finally, in order to remove the hypothesis on the density of A in the theorem above, the author allows unions of subgroups to enter the picture, but we shall not state the full result here.
The paper, and in particular the introduction, is beautifully written. It draws on a number of techniques from [*B. Green* and *T. Sanders*, Ann. Math. (2) 168, No. 3, 1025–1054 (2008; Zbl 1170.43003)], including approximately 0,1-valued functions and so-called Bourgain systems, and employs an iterative method of proof. *Julia Wolf (Palaiseau)*

Ulcigrai, Corinna

Spellings: Ulcigrai, Corinna [9] Ulcigrai, C. [1]
Author-Id: ulcigrai.corinna
Publications: 10 including 7 Journal Article(s)

MSC 2010

10	**37** · Dynamical systems and ergodic theory
4	**11** · Number theory
2	**60** · Probability theory and stochastic processes

Journals

2	Ergodic Theory and Dynamical Systems
1	Annals of Mathematics. Second Series
1	Journal of Fixed Point Theory and Applications
1	Journal of Modern Dynamics
1	Letters in Mathematical Physics

more ...

Co-Authors

4	Sinai, Yakov G.
2	Smillie, John
1	Bufetov, Alexander I.
1	Sinai, Ya.G.

Publication Years

1230.37021
Smillie, John; Ulcigrai, Corinna
Beyond Sturmian sequences: coding linear trajectories in the regular octagon.
Proc. Lond. Math. Soc. (3) 102, No. 2, 291-340 (2011).

The symbolic coding of a linear trajectory in a regular $2n$-gon, where opposite sides are identified, keeps track of the sequence of sides hit by the trajectory. For $n = 2$, the non-periodic cutting sequences are exactly the Sturmian sequences. In the present paper, non-periodic cutting sequences are characterized for the case $n \geq 3$ in terms of a derivation operator and a coherence condition. Here, derivation means that only sandwiched letters are kept, i.e., letters L preceded and followed by the same letter L'.
Successive derivations and normalizations of the cutting sequence yield a $2n$-gon Farey expansion (or additive continued fraction expansion) of the angle of the linear trajectory. On the other hand, the continued fraction expansion gives a sequence of substitution operations that generate the cutting seqeunces of trajectories with that slope. In the case of the octagon, a direction has "terminating" Farey expansion if and only if it is in $\mathbb{Q}(\sqrt{2})$. This is similar to the case $n = 2$, where terminating Farey expansions correspond to rational numbers. The factor complexity, i.e., the number of different words of length k, of a cutting sequence is bounded by $(n-1)k+1$, and it is equal to $(n-1)k+1$ when the direction is non-terminating.

The algorithm described by the authors can be understood in terms of renormalization of the $2n$-gon translation surface by elements of the Veech group; see also [the authors, Contemp. Math. 532, 29–65 (2010; Zbl 05885281)].

Wolfgang Steiner (Sydney)

Zbl 1151.37010
Sinai, Yakov G.; Ulcigrai, Corinna
Renewal-type limit theorem for the Gauss map and continued fractions.
Ergodic Theory Dyn. Syst. 28, No. 2, 643-655 (2008).

It is known that, for any $\alpha \in (0,1) \setminus \mathbb{Q}$, the continued fraction expansion of α is given by

$$\alpha = \cfrac{1}{\alpha_1 + \frac{1}{\alpha_2 + \frac{1}{\cdots}}} = [\alpha_1, \alpha_2, \ldots, \alpha_n, \ldots],$$

where $\alpha_n \in \aleph_+$ are the entries of the continued fraction and $\{p_n/q_n\}_{n\in\aleph_+}$ are the convergents of α. Let also denote by $\mathcal{J}$ the Gauss map, that is the transformation on $(0,1)$ given by $\alpha \to \mathcal{J}(\alpha) = \{1/\alpha\}$, where $\{.\}$ denotes the fractional part. For a given $\alpha \in (0,1) \setminus \mathbb{Q}$ and $R > 0$ let us denote by q_{n_r} the first denominator of the convergents of α which exceeds R.

The aim of the present paper is to introduce and prove a renewal-type limit theorem for the Gauss map and continued fractions of the above-mentioned form. More precisely, the authors managed to find out that the ratio q_{n_R}/R has a limiting distribution as R tends to infinity, where the existence of the limiting distribution uses mixing of a special flow over the natural extension of the Gauss map. *Cryssoula Ganatsiou (Larissa)*

Trélat, Emmanuel

Spellings: Trélat, Emmanuel [28] Trélat, E. [16] Trelat, E. [5]
Author-Id: trelat.emmanuel
Publications: 49 including 2 Book(s) and 41 Journal Article(s)

MSC 2010

26	**49** · Calculus of variations and optimal control; optimization
22	**93** · Systems theory; control
13	**53** · Differential geometry
9	**35** · Partial differential equations (PDE)
9	**70** · Mechanics of particles and systems

more ...

Journals

5	SIAM Journal on Control and Optimization
4	Journal of Dynamical and Control Systems
3	European Series in Applied and Industrial Mathematics (ESAIM): Control, Optimization and Calculus of Variations
2	Communications on Pure and Applied Analysis
2	Comptes Rendus. Mathématique. Académie des Sciences, Paris

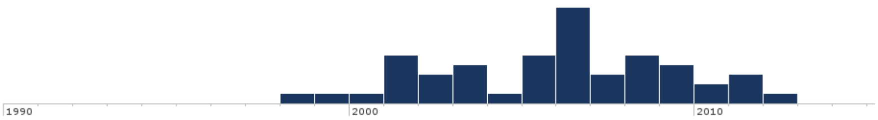
more ...

Co-Authors

7	Bonnard, Bernhard
4	Bonnard, Bernard
4	Chitour, Yacine
4	Coron, Jean-Michel
3	Caillau, Jean-Baptiste

more ...

Publication Years



Zbl 1123.49014
Bonnard, Bernard; Caillau, Jean-Baptiste; Trélat, Emmanuel
Second order optimality conditions in the smooth case and applications in optimal control.
ESAIM, Control Optim. Calc. Var. 13, No. 2, 207-236 (2007).

The authors deal with the control system

$$\dot{x}(t) = f(x(t), u(t)),$$

where $f : M \times N \to TM$ is a smooth function with manifolds M, N of dimensions m and n respectively. If M_0 and M_1 are two subsets of M, then $\mathcal{U}$ is the set of admissible controls such that the associated trajectories steer the system form an initial point of M_0 to a final point in M_1. For such controls u the cost function is defined by

$$C(t_f, u) = \int_0^{t_f} f_0(x(t), u(t))\, dt,$$

where $f_0 : M \times N \to R$ is smooth. Applying the Pontryagin maximum principle one receives the Hamiltonian system for the trajectory $x(.)$ connected with an optimal control $u \in \mathcal{U}$, the adjoint vector $p(.)$ and a nonpositive number p^0 satisfying $(p(.), p^0) \neq (0, 0)$. An extremal $(x(.), p(.), p^0, u(.))$ is called normal if $p^0 < 0$ and abnormal if $p^0 = 0$. The authors present algorithms to compute the first conjugate time along a smooth extremal curve, where the trajectory ceases to be optimal. The article contains a review of second order optimality conditions. The computations are related to a test of positivity of the intrinsic second order derivative or a test of singularity of the extremal flow. Their algorithm is applied to the minimal time problem and to the attitude control problem of a rigid spacecraft. The algorithm involves both normal and abnormal cases. *Igor Bock (Bratislava)*

Zbl 1112.49001
Trélat, Emmanuel
Optimal control. Theory and applications.
(Contrôle optimal. Théorie et applications.) (French)
Mathématiques Concrètes.
Paris: Vuibert (ISBN 2-7117-7175-X/pbk). vi, 246 p. EUR 30.00 (2005).

This work of Emmanuel Trélat, published in 2005, is an excellent new text book that guides the interested reader into the world of optimal control. To be more precise, it is the optimal control of ordinary differential equations (ODEs) or of systems of ODEs, that this work is concerned with. In literature, there are many possible approaches to optimal control yet: theoretical ones which stay inside of the frames of convex or nonsmooth analysis and optimization, of functional analysis or topology, or numerical ones with a focus on the resolution of the mathematical task, or practical ones which show the problems applied at concrete real-word problems.
The present book is a classical one in so far as it carefully presents and classifies the optimal control problems, presents the analytical problems related and tackles them. Here the underlying perspective is a one from optimization theory and the theory of dynamical systems, including the algorithmical and simulation parts. But the book is also a modern one in so far as it addresses real-world problems, it presents the whole circle from these motivations to the numerical solution. All of this lets the book become a self-contained entity. It is well-structured, equipped with an appropriate design; both writing style and layout are "light" and inviting. Indeed the whole books invites to do exercises, to apply the knowledge obtained to small tasks and problems, and beyond this text book, to become an expert, practitioner and, maybe, researcher in the areas of optimal control. This book could become a stable basis for more advanced studies done in the future. The proofs are given carefully. Many nice photos, illustrations, codes and tables make the reading and studying of this book a pleasure and interesting. The applications range between, e.g., mechanics and aerodynamics to chemical engineering and ecology.
This book begins with notations, opening remarks and an introduction. Then, it contains of three parts. Part 1 is about optimal control of linear systems: controllability, time optimality and linear-quadratic theory. Part 2 is on non-

linear optimal control theory: definitions, preliminaries and problem presentation of optimal control, Pontryagin's maximum principle, Hamilton-Jacobi theory and numerical methods. Part 3 contains appendices about tools from linear algebra, the theorem of Cauchy-Lipschitz, modeling of a linear control system, stabilization and observability of control systems.
One can truly thank and congratulate the author to this valuable contribution to learning and teaching, and to a support and stimulation for scientific enterprises. Indeed, even those who will be concerned with optimal control of discrete systems, of partial differential equations or stochastic differential equations can benefit from this book, from the foundations given by it and in view of various similarities with those other areas of optimal control.
One final word on the language: Today, most text books are written in English, the present one in French. There reviewer is convinced that there should not remain a big obstacle for those who do not know French. Mathematics and its terminology are so international that the reader will get used to the language after some days, he or she will enjoy reading and gain a lot from this precious work!

Gerhard-Wilhelm Weber (Ankara)

Zbl 05994874
Hogendijk, Jan P.
Two beautiful geometrical theorems by Abū Sahl Kūhī in a 17th century Dutch translation
Tarikh-e Elm, Iran. J. Hist. Sci. 6, 1-36 (2008).

The theorem of the arbelos (or shoemaker's knife) refers to Lemma 5 of the *Book of Lemmas*, a book attributed to Archimedes by Thābit ibn Qurra. This theorem can be combined with al-Kūhī's two beautiful generalizations alluded to in the title of the paper under review in one theorem as follows: *Let C, D be any two points on the open line segment AB, let Ω, Ω_1, Ω_2 be the semicircles having diameters AB, AC, BD and drawn on the same side of AB, and let Λ be the radical axis of Ω_1, Ω_2. Then the circle touching Ω, Ω_1, Λ is equal to the circle touching Ω, Ω_2, Λ.*

The original version of Archimedes corresponds to the case when $C = D$, while the two generalizations of al-Kūhī correspond to the cases when C lies between A, D and when D lies between A, C. These two generalizations of al-Kūhī were preserved in the commentary on the *Book of Lemmas* written by Nasawī in the eleventh century and were later included in the edition of the *Middle Books* written by al-Tūsī in the thirteenth century. They later appeared in Latin and then in Dutch translations in the seventeenth century. The paper under review compares al-Tūsī's Arabic version with the Dutch translation. Among other things, the comparison shows that while the Dutch version contains a serious mistake in the proof, it contains an interesting, and seemingly original, addition pertaining to the construction of the radical axis of two non-intersecting circles.

The paper contains the Arabic, English and Dutch texts of these generalizations as well as of their proofs. An interesting feature of the proofs are intermediary lemmas that explicitly state and prove, in a manner very simple and accessible to Euclid, what is equivalent to the fact that *the altitudes of a triangle are concurrent*. Such a fact is only *implicit* in Archimedes' proof of the original arbelos theorem, where it is referred to as *a property of triangles*; see line 4, page 306 of [The works of Archimedes. Edited in modern notation, with introductory chapters by Thomas L. Heath. Reprint of the 1897 original. New York: Dover Publications (2009; Zbl 1200.01042)]. None of this answers our curiosity as to why such a simple and natural concurrence theorem is never mentioned in Euclid's *Elements*!

Archimedes' arbelos theorem has been generalized by many as can be seen from the series of papers written by *H. Okumura* and *M. Watanabe* in [Forum Geom.] and the references therein. It has even been generalized to dimension three by *S. Abu-Saymeh* and *M. Hajja* in [Result. Math. 52, No. 1–2, 1–16 (2008; Zbl 1147.51011)]. However, this reviewer does not seem to have seen al-Kūhī's generalizations anywhere (except in the afore-mentioned book of Heath [p. 307, first paragraph]). The obscurity of these nice generalizations and their suitability for a first course of geometry make the paper under review interesting, not only to students of the history of mathematics, but to students of geometry as well.

Mowaffaq Hajja (Irbid)

Zbl 555.01001
Hogendijk, Jan P.
Greek and Arabic constructions of the regular heptagon.
Arch. Hist. Exact Sci. 30, 197-330 (1984).

The project of research planned for this extensive paper might be well chosen some century ago, but appearing after the second half of the 20th century it could only lead to disappointments. With the discovery of the regular polygons, treated and computed in the 2nd millennium B.C., it is clear that these studies had to be based on the isosceles trapezium, the next step after "Pythagoras" to the relation of "Ptolemy". It replaces all which can be done by "trigonometry", governed by the rule that the square of the diagonal is equal to the sum of the square of the equal sides and the product of the parallel sides.
The fact that al-Birûni asked to show that the side of the enneagon is a root of a cubic equation demonstrates that the equations were looked for a priori. In the babylonian "geometry of the plummet" [cf. the reviewer, *Simon Stevin* 33, 38-60 (1950; Zbl 0089.003)] the problem of the heptagon was treated. The heptagon has three determining quantities, a,d,D viz. the length of the side and two diagonals. Specifying the isosceles trapezia by the sequence of numbered vertices one obtains four quadratic relations: $d^2 = a^2 + aD$, (1234); $D^2 = a^2 + dD$, (1245); $D^2 = d^2 + ad$, (1246) and from these by additions and subtractions a fourth relation, a ptolemaic relation: $dD = ad + aD$ (4).
The first and the third relation are those mainly quoted in a "lemma of Archimedes". The present reviewer wrote since 1959 the quotation marks indicating his opinion that this attribution is not correct. A transversal in a square can be used to solve cubic equations. We see from completing rectangles - PESD, ERQT - that $z^2 = y(y + x)$, a relation of the form (1), and that the double of the area EAB is $ay = S_1$. With $BF = k$ we have $ay = kz$ and two times the S_2-area CFG is $x(a - k) = ax(z - y)/z$. This leads for the condition $pS_1 = S_2$ to $pyz = xz - zy$, which is all we need to see that a second equation for the heptagon arises for $p = 1$ in (4), whence in x,y,z we have quantities D,a,d of the heptagon.
In such homogeneous systems one can choose one "unknown" and is left with quadratic equations, which can be solved by intersection of conics, and two conics need a third in order to eliminate duely a "parasitic" solution. Through three points pass ∞^2 conics and thus there are ∞^4 possible choices for a fitting pair of conics. One can therefore only meet with "particular cases", and the more cases one has treated the "more general" the work is considered. We think that the blaming by the author of R. Rashed is unjust. The author's remark that the problem comes out to determine sin 180∘/7 is certainly futile. In fact till the time of Huygens and De Sluse one had

no idea of pencils and nets of conics and curves, and one did not combine "geometrical equations" in reductions.

It must be left to the interested reader to find - what the author neglected to indicate - how just specialising a chosen quantity the equations of the conics used by the Arabs are just a pair of the above given tetrad. This is unnecessarily complicated by the author's deliberately drawing mirror-images of some figures in the manuscript. This causes an, impossible for the Arabs, coordinate transformation $x = -x'$, $y = y'$.

As the author did not treat the possibilities in the several cases he mentions, we have to indicate here that al-Kuhi - just as Menaichmos - did not see the fact that the points of intersection of his first, and partial, hyperbola and parabola are concircular.

By the work of the author we find put together lists of manuscripts and verbatim translations on solutions, quarrels between young geometers on errors, mistakes and priority, and have instead of the old about seven published solutions some 12 combinations of conics out of the ∞^4 making the solution "general" and not only some particular case. This is hardly a contribution to the problem-history of mathematics itself. One might ask whether one should waste his time on such a project, just for completing some sections of not yet edited manuscripts, on an already well known situation and the level of knowledge of the period. *E.M. Bruins*