The $r$-acyclic chromatic number of planar graphs.


Summary: A vertex coloring of a graph $G$ is $r$-acyclic if it is a proper vertex coloring such that every cycle $C$ receives at least $\min\{|C|, r\}$ colors. The $r$-acyclic chromatic number $a_r(G)$ of $G$ is the least number of colors in an $r$-acyclic coloring of $G$. Let $G$ be a planar graph. By Four Color Theorem, we know that $a_1(G) = a_2(G) = \chi(G) \leq 4$, where $\chi(G)$ is the chromatic number of $G$. O. V. Borodin [Discrete Math. 25, 211–236 (1979; Zbl 0406.05031)] proved that $a_3(G) \leq 5$. However when $r \geq 4$, the $r$-acyclic chromatic number of a class of graphs may not be bounded by a constant number. For example, $a_4(K_{2,n}) = n + 2 = \Delta(K_{2,n}) + 2$ for $n \geq 2$, where $K_{2,n}$ is a complete bipartite (planar) graph. In this paper, we give a sufficient condition for $a_r(G) \leq r$ when $G$ is a planar graph. In precise, we show that if $r \geq 4$ and $G$ is a planar graph with $g(G) \geq \frac{10r-4}{3}$, then $a_r(G) \leq r$. In addition, we discuss the 4-acyclic colorings of some special planar graphs.

Keywords: acyclic coloring; planar graph; girth