Perturbation analysis for the Darcy problem with log-normal permeability.

Summary: We study the single-phase flow in a saturated, bounded heterogeneous porous medium. We model the permeability as a log-normal random field. We perform a perturbation analysis, expanding the solution in Taylor series. The approximation properties of the Taylor polynomial are studied, and the local convergence of the Taylor series is proved. With a counterexample we show that, in general, the Taylor series is not globally convergent to the stochastic solution as the polynomial degree goes to infinity. Nevertheless, for small variability of the permeability field and low degree of the Taylor polynomial, the perturbation approach is feasible and provides a good approximation of both the stochastic solution and the statistical moments of the stochastic solution. We derive an upper bound on the norm of the residual of the Taylor series, which predicts the optimal degree of the Taylor polynomial to consider. The upper bound is quite pessimistic. In the simple case of a permeability field described by only one random variable, we show numerically that a simple “tuning” of the upper bound, which uses estimates of the growth of the derivatives, provides sharp bounds.

Keywords: perturbation technique; uncertainty quantification; elliptic PDE with random coefficient; log-normal distribution

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