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**Janet's algorithm.**
Bull. Math. Soc. Sci. Math. Roum., Nouv. Sér. 51(99), No. 1, 11-19 (2008).

Let $S = K[x_1, \ldots, x_n]$ be a polynomial ring over a field $K$ and $M$ a finitely generated $\mathbb{Z}^n$-graded $S$-module. A Stanley space is a $\mathbb{Z}^n$-graded $K$-subspace of $M$ of the form $uK[Z]$, where $u$ is a $\mathbb{Z}^n$-homogeneous element of $M$ and $Z$ is a subset of $\{x_1, \ldots, x_n\}$. A Stanley decomposition of $M$ is a presentation of $M$ as a finite direct sum of Stanley subspaces: $M = \bigoplus_{i=1}^m u_i K[Z_i]$. In the paper under review, the author describes Janet's algorithm, which originates in the work of *M. Janet* [Ann. de l'Éc. Norm. (3) 41, 27–65 (1924; JFM 50.0321.03)] and produces a Stanley decomposition in the case where $M = I$ or $M = S/I$, $I$ a monomial ideal of $S$. It is a recursive procedure on the number $n$ of indeterminates. Let $S' := K[x_1, \ldots, x_{n-1}]$ and, for every $k \geq 0$, let $I_k := (I : x_n^k) \cap S'$. $I_k$ is an ideal of $S'$ and $I_0 \subseteq I_1 \subseteq \cdots \subseteq I_k \subseteq \ldots$ hence, for $k >> 0$, $I_k = I_{k+1} = \cdots$. Let $\beta$ be the least $k$ for which this happens and let $\alpha$ be the least $k$ for which $I_k \neq (0)$. Then:

$$I = \bigoplus_k x_n^k I_k = (\bigoplus_{\alpha \leq k < \beta} x_n^k I_k) \bigoplus x_n^\beta K[x_n] I_\beta,$$
$$S/I = \bigoplus_k x_n^k (S'/I_k) = (\bigoplus_{\alpha \leq k < \beta} x_n^k (S'/I_k)) \bigoplus x_n^\beta K[x_n](S'/I_\beta)$$

hence, if $I_k$ and $S'/I_k$ admit Stanley decompositions over $S'$, $I$ and $S/I$ admit Stanley decompositions over $S$. When $I$ is a squarefree monomial ideal, Janet's algorithm produces a squarefree Stanley decomposition, which means that $u_i$ is a squarefree monomial and $u_i \in K[Z_i]$, $\forall i$. The author also presents a variant of Janet's algorithm producing a partition of a simplicial complex $\Delta$, i.e., a presentation of the simplicial complex as a disjoint union of subsets of the form $\{H \in \Delta \mid F \subseteq H \subseteq G\}$ for some $F, G \in \Delta$. He shows, by an example, that Janet's algorithm does not necessarily produce a nice partition (which means that the $G$s are facets of $\Delta$) even if the simplicial complex admits one. *Iustin Coanda (Bucureşti)*

*Keywords:* monomial ideal; Stanley decomposition; partition of a simplicial complex

 