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Uniform controllability properties for space/time-discretized parabolic equations.

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Summary: This article is concerned with the analysis of semi-discrete-in-space and fully-discrete approximations of the null controllability (and controllability to the trajectories) for parabolic equations. We propose an abstract setting for space discretizations that potentially encompasses various numerical methods and we study how the controllability problems depend on the discretization parameters. For time discretization we use Θ-schemes with $\Theta \in [\frac{1}{2}, 1]$. For the proofs of controllability we rely on the strategy introduced by *G. Lebeau* and *L. Robbiano* [Commun. Partial Differ. Equations 20, No. 1–2, 335–356 (1995; Zbl 0819.35071)] for the null-controllability of the heat equation, which is based on a spectral inequality. We obtain relaxed uniform observability estimates in both the semi-discrete and fully-discrete frameworks, and associated uniform controllability properties. For the practical computation of the control functions we follow J.-L. Lions' Hilbert's uniqueness method strategy, exploiting the relaxed uniform observability estimate. Algorithms for the computation of the controls are proposed and analysed in the semi-discrete and fully-discrete cases. Additionally, we prove an error bound between the fully discrete and the semi-discrete control functions. This bound is however not uniform with respect to the space discretization. The theoretical results are illustrated through numerical experimentations.

Keywords: semi-discrete-in-space and fully-discrete approximations; null controllability; parabolic equations; time discretization; relaxed uniform observability estimates; J.-L. Lions' Hilbert's uniqueness method; error bound
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