Proving inequalities and solving global optimization problems via simplified CAD projection.

Summary: Let $x_n = (x_1, \ldots, x_n)$ and $f \in \mathbb{R}[x_n, k]$. The problem of finding all $k_0$ such that $f(x_n, k_0) \geq 0$ on $\mathbb{R}^n$ is considered in this paper, which obviously takes as a special case the problem of computing the global infimum or proving the semi-definiteness of a polynomial. For solving the problems, we propose a simplified Brown-McCallum’s CAD projection operator, Nproj, of which the projection scale is always no larger than that of Brown-McCallum’s. For many problems, the projection scale is much smaller than that of Brown-McCallum’s. As a result, the lifting phase is also simplified. Some new algorithms based on Nproj for solving those problems are designed and proved to be correct. Comparison to some existing tools on some examples is reported to illustrate the effectiveness of our new algorithms.

**Keywords:** CAD projection; global optimization; semi-definiteness; polynomials

doi:10.1016/j.jsc.2015.02.007