A complete solution to spectrum problem for five-vertex graphs with application to traffic grooming in optical networks.

Summary: A $G$-design of order $n$ is a decomposition of the complete graph on $n$ vertices into edge-disjoint subgraphs isomorphic to $G$. Grooming uniform all-to-all traffic in optical ring networks with grooming ratio $C$ requires the determination of graph decompositions of the complete graph on $n$ vertices into subgraphs each having at most $C$ edges. The drop cost of such a grooming is the total number of vertices of nonzero degree in these subgraphs, and the grooming is optimal when the drop cost is minimum. The existence spectrum problem of $G$-designs for five-vertex graphs is a long standing problem posed by J.-C. Bermond et al. [Ars Comb. 10, 211–254 (1980; Zbl 0454.05053)], which is closely related to traffic groomings in optical networks. Although considerable progress has been made over the past 30 years, the existence problems for such $G$-designs and their related traffic groomings in optical networks are far from complete. In this paper, we first give a complete solution to this spectrum problem for five-vertex graphs by eliminating all the undetermined possible exceptions. Then, we determine almost completely the minimum drop cost of 8-groomings for all orders $n$ by reducing the 37 possible exceptions to 8. Finally, we show the minimum possible drop cost of 9-groomings for all orders $n$ is realizable with 14 exceptions and 12 possible exceptions.

Keywords: graph decomposition; $G$-design; optical networks; traffic grooming; wavelength-division multiplexing

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