Fuzzy responses and bifurcations of a forced Duffing oscillator with a triple-well potential.

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Summary: Responses and bifurcations of a forced triple-well potential system with fuzzy uncertainty are studied by means of the Fuzzy Generalized Cell Mapping (FGCM) method. A rigorous mathematical foundation of the FGCM is established as a discrete representation of the fuzzy master equation for the possibility transition of continuous fuzzy processes. The FGCM offers a very effective approach for solutions to the fuzzy master equation based on the min-max operator of fuzzy logic. A fuzzy response is characterized by its topology in the state space and its possibility measure of membership distribution functions (MDFs). A fuzzy bifurcation implies a sudden change both in the topology and in the MDFs. The response topology is obtained based on the qualitative analysis of the FGCM involving the Boolean operation of 0 and 1. The MDFs are determined by the quantitative analysis of the FGCM with the min-max calculations. With an increase of the intensity of fuzzy noise, noise-induced escape from each of the potential wells defines two types of bifurcations, namely catastrophe and explosion. This paper focuses on the evolution of transient and steady-state MDFs of the fuzzy response. As the intensity of fuzzy noise increases, steady-state MDFs cover a bigger area in the state space with higher membership values spreading out to a larger area. The previous conjectures are further confirmed that steady-state MDFs are dependent on initial possibility distributions due to the nonsmooth and nonlinear nature of the min-max operation. It is found that as time goes on, transient MDFs spread around three potential wells. The evolutionary orientation of transient MDFs aligns with unstable invariant manifolds leading to stable invariant sets. Two examples of additive and multiplicative fuzzy noise are given.

Keywords: fuzzy uncertainty; possibility measure; fuzzy response; membership distribution function; generalized cell mapping
doi:10.1142/S0218127415500054