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Nonstandard construction of Brownian motion and martingales on Lie groups.
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The author's aims is to define the Brownian motion and also one and two parameter G-martingales on the Lie groups by means of nonstandard analysis. The work consists of the introduction, two chapters, and appendix. In the introduction the history of the problem is presented. Chapter 1 begins with a brief exposition of the main principles of the nonstandard analysis. Then a nonstandard representation of the Stratonovich integral is given and its standard part is shown to correspond to the usual Stratonovich integral. The author formulates the fundamental methodology of the work in the following manner: to begin with a usual definition, then find a lifting, i.e., a standard notion which verifies the same properties but transferred to the nonstandard universe, then deal with this notion in the nonstandard universe, and finally going back to the standard universe by means of the standard map. It is proved that for a real manifold M there exists a nonstandard manifold N such that $M \subseteq N \subseteq {}^*M$, ${}^*M$ being the transfer of M to the nonstandard universe. This allows to find liftings of semi-martingales on Lie groups. Then a definition of Brownian motion on Lie groups as an internal stochastic process is given. It is shown that this stochastic process satisfies some differential equation which characterizes the Brownian motion on Lie groups. The nonstandard definition of Wiener measure on Lie algebras is formulated. Then by means of Itô's map the notion of nonstandard representation of the Wiener measure on $P_e(G)$, where G is a Lie group, is obtained. Chapter 2 contains the main results of the work. Here, the notion of internal two parameter local martingale is introduced and results on liftings and standard parts of local martingales are proved. The existence of local martingales is shown by means of liftings. It is shown that the standard parts of several kinds of nonstandard two parameter martingales are martingales of the same kind. In the last section of the work a nonstandard version of the stochastic integral with respect to a two parameter nonstandard S-square integrable local martingale is presented. It is proved that for each kind of standard martingales there exist liftings that are nonstandard martingales of the same kind. A nonstandard representation of two parameter G-i-martingales are given. In the Appendix a survey of the basic notions and theorems of the nonstandard analysis are presented. *G.Pestov (Tomsk)*

Keywords: nonstandard analysis; nonstandard representation; Stratonovich integral; liftings of semi-martingales on Lie groups; Brownian motion on Lie groups; local martingales