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Extensions of Büchi’s higher powers problem to positive characteristic.

Summary: Büchi’s $n$th power problem on $\mathbb{Q}$ asks whether there exist an integer $M$ such that the only monic polynomials $F \in \mathbb{Q}[X]$ of degree $n$ satisfying that $F(1), \ldots, F(M)$ are $n$th power rational numbers, are precisely of the form $F(X) = (X + c)^n$ for some $c \in \mathbb{Q}$. In this paper, we study analogs of this problem for algebraic function fields of positive characteristic. We formulate and prove an analog (indeed, such a formulation for $n > 2$ was missing in the literature due to some unexpected phenomena), which we use to derive some definability and undecidability consequences. Moreover, in the case of characteristic zero, we extend some known results by improving the bounds for $M$ (from quadratic on $n$ to linear on $n$).

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