Chen, Wen; Liang, Yingjie; Hu, Shuai; Sun, Hongguang
Fractional derivative anomalous diffusion equation modeling prime number distribution.

Summary: This study suggests that the power law decay of prime number distribution can be considered a sub-diffusion process, one of typical anomalous diffusions, and could be described by the fractional derivative equation model, whose solution is the statistical density function of Mittag-Leffler distribution. It is observed that the Mittag-Leffler distribution of the fractional derivative diffusion equation agrees well with the prime number distribution and performs far better than the prime number theory. Compared with the Riemann’s method, the fractional diffusion model is less accurate but has clear physical significance and appears more stable. We also find that the Shannon entropies of the Riemann’s description and the fractional diffusion models are both very close to the original entropy of prime numbers. The proposed model appears an attractive physical description of the power law decay of prime number distribution and opens a new methodology in this field.

Keywords: prime number distribution; fractional calculus; anomalous diffusion; Mittag-Leffler distribution; entropy