
io-port 01586421**Barr, Michael****On *-autonomous categories of topological vector spaces.**

Cah. Topologie Géom. Différ. Catég. 41, No.4, 243-254 (2000).

TVS is the category of locally convex topological vector spaces over \mathbb{K} , where \mathbb{K} is the field of real or complex numbers. The full subcategory of **TVS** generated by the spaces E in **TVS** which carry the weak topology $\sigma(E, E')$, E' the (topological) dual of E , is denoted by \mathbf{T}_w . The full subcategory of **TVS** generated by the spaces E in **TVS** with the Mackey topology $\mu(E, E')$ is denoted by \mathbf{T}_m . The author proves (Theorem 3.1) that \mathbf{T}_w and \mathbf{T}_m are *-autonomous categories [cf. *M. Barr*, “*-Autonomous categories”, Lect. Notes Math. 752 (1979; Zbl 0415.18008)] and that they are isomorphic. This is done by showing that both categories are equivalent to the category **chu** of separated and extensional pairs (loc. cit.) $\langle E, E' \rangle$, which are also known as dual pairings [cf. *H. Jarchow*, “Locally Convex Spaces”, Math. Leitfäden (1981; Zbl 0466.46001)] or dualities [cf. *H. H. Schaefer*, “Topological Vector Spaces”, Graduate Texts (1980; Zbl 0435.46002)] in the literature. The author uses the characterization of the Mackey topology of a space E as the finest topology compatible with the pair $\langle E, E' \rangle$ which is given by the Mackey-Arens theorem [cf. e.g. *H. Jarchow*, loc. cit.]. He gives an interesting categorical proof for the existence of this finest compatible topology (4.3). This proof corresponds to the existence proof in general topology for the supremum of all the topologies on E compatible with $\langle E, E' \rangle$ in the complete lattice of all topologies on E [cp. e.g. *N. Bourbaki*, “Topologie Générale”, Livre III, Paris (1953; Zbl 0050.38903)] or to the proof of this supremum for a topological category in categorical topology [ep. e.g. *O. Wyler*, General Topology Appl. 1, 17-28 (1971; Zbl 0215.51502)], respectively.

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