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io-port 00720232
Glynn, Peter W.
Poisson's equation for the recurrent $M/G/1$ queue.
Adv. Appl. Probab. 26, No.4, 1044-1062 (1994).

The primary objective of the paper is to study the behavior of time averages of the form $r_n(f) = \frac{1}{n}\sum_{k=0}^{n-1} f(W_k)$, where f is a real-valued function defined on R^+, W_k is waiting time sequence for the $M/G/1$ queue. When the utilization factor $\rho < 1$, $r_n(f)$ has a limit $r(f)$ as $n \to \infty$, and the Pollaczek-Khinchin formula identifies this limit. This limit is used as an approximation to $r_n(f)$ and it is of some interest to obtain results giving the rate at which $r_n(f)$ converges to $r(f)$. Here formulas are derived which describe the fluctuations of $r_n(f)$ about its limit. The concrete approach is to express the process $S_n(f) \equiv nr_n(f)$ in terms of a martingale sequence. The martingale is constructed by solving Poisson's integral equation. Using random walk theory it is possible to solve this equation explicitly. The martingale structure of $S_n(f)$ then easily leads to limit theorems of forms

$$Er_n(f) = r(f) + \frac{1}{n}c(f) + o(1/n) \quad \text{and} \quad n^{1/2}(r_n(f) - r(f)) \Rightarrow \sigma(f)N(0,1)$$

as $n \to \infty$, where $\Rightarrow$ denotes weak convergence, $N(0,1)$ is a standard normal variable.

L.Lakatos (Budapest)

Keywords: behavior of time averages; Pollaczek-Khinchin formula; Poisson's integral equation; random walk theory; limit theorems; weak convergence
doi:10.2307/1427904