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On the complexity of some hierarchical structured matrix algorithms.

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Summary: In recent years, hierarchical structured matrices have been widely used in fast solutions of integral equations, partial differential equations, structured matrix (such as Toeplitz) problems, companion eigenproblems, etc. In this paper, we systematically study the complexity of some hierarchical structured matrix algorithms, in terms of hierarchically semiseparable (HSS) matrices. Several important aspects are considered. We perform detailed complexity analysis for some typical HSS algorithms, with the aid of certain graph techniques. This analysis helps us to provide some significant improvements to classical HSS methods. One improvement is to propose more efficient HSS construction and solution algorithms. Another improvement is a recompression procedure which reorthonormalizes some HSS generators and converts noncompact HSS forms to compact ones. A precise theoretical justification of the compactness is also given. The third improvement is to relax the rank requirement in HSS operations. Unlike many classical HSS methods where the appropriate off-diagonal (numerical) ranks are often required to be bounded, we allow the ranks to increase. Certain general rank patterns are proposed, so that similar performance can be achieved with the maximum rank unbounded. These improvements significantly enhance both the efficiency and the applicability of HSS methods. Numerical examples from some applications are included to support the analysis.

Keywords: matrix factorization; hierarchical structured matrix algorithms; hierarchically semiseparable matrices; graph techniques; numerical examples
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