Effect of graph structure on the limit sets of threshold dynamical systems.

Summary: We study the attractor structure of standard block-sequential threshold dynamical systems. In a block-sequential update, the vertex set of the graph is partitioned into blocks, and the blocks are updated sequentially while the vertices within each block are updated in parallel. There are several notable previous results concerning the two extreme cases of block-sequential update: (i) sequential and (ii) parallel. While parallel threshold systems can have limit cycles of length at most two, sequential systems can have only fixed points. However, Goles and Montealegre [5] showed the existence of block-sequential threshold systems that have arbitrarily long limit cycles. Motivated by this result, we study how the underlying graph structure influences the limit cycle structure of block-sequential systems. We derive a sufficient condition on the graph structure so that the system has only fixed points as limit cycles. We also identify several well-known graph families that satisfy this condition.

Keywords: graph dynamical systems; generalized cellular automata; threshold functions; block decomposition

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