Summary: Let $D$ be a digraph with vertex set $V(D)$. A vertex $x$ is a $k$-king of $D$, if for every $y \in V(D)$, there is an $(x, y)$-path of length at most $k$. A subset $N$ of $V(D)$ is $k$-independent if for every pair of vertices $u, v \in N$, we have $d(u, v) \geq k$ and $d(v, u) \geq k$; it is $l$-absorbent if for every $u \in V(D) - N$ there exists $v \in N$ such that $d(u, v) \leq l$. A $(k, l)$-kernel of $D$ is a $k$-independent and $l$-absorbent subset of $V(D)$. A $k$-kernel is a $(k, k - 1)$-kernel. A digraph $D$ is $k$-quasitransitive, if for any path $x_0x_1 \ldots x_k$ of length $k$, $x_0$ and $x_k$ are adjacent. In this article, we will prove that a $k$-quasitransitive digraph with $k \geq 4$ has a $k$-king if and only if it has a unique initial strong component and the unique initial strong component is not isomorphic to an extended $(k + 1)$-cycle $C[E_0, E_1, \ldots, E_k]$ where each $E_i$ has at least two vertices. Using this fact, we show that every strong $k$-quasitransitive digraph has a $(k + 1)$-kernel.

Keywords: quasitransitive digraph; $k$-quasitransitive digraph; $k$-king; $k$-kernel

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