
io-port 05876326**Shalit, Orr Moshe****Stable polynomial division and essential normality of graded Hilbert modules.**

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W. Arveson [J. Oper. Theory 54, No. 1, 101–117 (2005; Zbl 1107.47006)] conjectured that every graded submodule M of $H_d^2 \otimes \mathbb{C}^r$ and its quotient $H_d^2 \otimes \mathbb{C}^r/M$ are p -essentially normal for $p > d$. This conjecture has been verified for modules generated by monomials [see *R. G. Douglas*, J. Oper. Theory 55, No. 1, 117–133 (2006; Zbl 1108.47030)] and also for principal modules as well as arbitrary modules in dimensions $d = 2, 3$ [cf. *K.-Y. Guo* and *K. Wang*, Math. Ann. 340, No. 4, 907–934 (2008; Zbl 1148.47005)]. In order to tackle this conjecture, the author introduces the stable division property for modules (and ideals); a normed module M over the ring of polynomials in d variables is said to possess the stable division property if it has a generating set $\{f_1, \dots, f_k\}$ such that every $h \in M$ can be written as $h = \sum_i a_i f_i$ for some polynomials a_i such that $\sum \|a_i f_i\| \leq C \|h\|$. He shows that when the algebra of polynomials in d variables is given the natural ℓ^1 -norm, then every ideal is linearly equivalent to an ideal that has the stable division property. He then proves that a module M that has the stable division property is p -essentially normal for $p > \dim(M)$, as conjectured by *R. G. Douglas* [“A new kind of index theorem”, in: Analysis, geometry and topology of elliptic operators. Papers of a workshop in honor of Krzysztof P. Wojciechowski on his 50th birthday, Roskilde, Denmark, May 20–22, 2005 (Hackensack, NJ: World Scientific), 369–382 (2006; Zbl 1140.47060)].

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