Summary: Let $G$ be a nontrivial connected graph with an edge-coloring $c : E(G) \rightarrow \{1, 2, \ldots, q\}$, $q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree $T$ in $G$ is a rainbow tree if no two edges of $T$ receive the same color. For a vertex subset $S \subseteq V(G)$, a tree that connects $S$ in $G$ is called an $S$-tree. The minimum number of colors that are needed in an edge-coloring of $G$ such that there is a rainbow $S$-tree for each $k$-subset $S$ of $V(G)$ is called the $k$-rainbow index of $G$, denoted by $r_x k(G)$. In this paper, we first determine the graphs of size $m$ whose 3-rainbow index equals $m$, $m - 1$, $m - 2$ or 2. We also obtain the exact values of $r_x 3(G)$ when $G$ is a regular multipartite complete graph or a wheel. Finally, we give a sharp upper bound for $r_x 3(G)$ when $G$ is 2-connected and 2-edge connected. Graphs $G$ for which $r_x 3(G)$ attains this upper bound are determined.

Keywords: rainbow tree; $S$-tree; $k$-rainbow index

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