Symmetric cubic graphs with solvable automorphism groups.

Summary: A cubic graph $\Gamma$ is $G$-arc-transitive if $G \leq \text{Aut}(\Gamma)$ acts transitively on the set of arcs of $\Gamma$, and $G$-basic if $\Gamma$ is $G$-arc-transitive and $G$ has no non-trivial normal subgroup with more than two orbits. Let $G$ be a solvable group. In this paper, we first classify all connected $G$-basic cubic graphs and determine the group structure for every $G$. Then, combining covering techniques, we prove that a connected cubic $G$-arc-transitive graph is either a Cayley graph, or its full automorphism group is of type $2^2$, that is, a 2-regular group with no involution reversing an edge, and has a non-abelian normal subgroup such that the corresponding quotient graph is the complete bipartite graph of order 6.

Keywords: $G$-arc-transitive cubic graph; connected $G$-basic cubic graphs
doi:10.1016/j.ejc.2014.10.008