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Sampling theorem for bandlimited Hardy space functions generated by Regge problem.

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A sampling theorem is established for a certain subspace of the Hardy space H^2_+ of functions F analytic in the upper half plane such that $\sup_{y>0}\int_{-\infty}^{\infty}|F(t+iy)|^2\,dt<\infty$. The authors define a space $\mathcal{H}$ of vector functions $U=(u_0(r),u_1(r))$ with norm $\|U\|^2_{\mathcal{H}}=\frac{1}{2}\int_0^\infty\big(|u_0'(r)|^2+\tilde{\rho}(r)|u_1(r)|^2\big)\,dr$ and respective *outgoing* and *incoming* spaces $\mathcal{D}_+$ and $\mathcal{D}_-$ defined by initial data (u_0,u_1) for solutions of the wave equation $u_{tt}(t,r)-(\tilde{\rho})^{-1}u_{rr}(t,r)=0$ on $(0,\infty)$ satisfying $u(t,0)=0$, $u(0,r)=u_0(r)$ and $u_t(0,r)=u_1(r)$, vanishing on $r-t\le a$ and $r+t\le a$ respectively. Here $\tilde{\rho}$ is an extension to $(0,\infty)$ of a density ρ defined on $[0,a)$ by setting $\tilde{\rho}(r)=1$ for $r>a$. The density ρ is required to satisfy certain regularity conditions near the endpoints $\{0,a\}$. Denote by $\{\lambda_n\}$ the eigenvalues of the wave equation and define the Blaschke product $S(z)=\prod_{n=-\infty}^{\infty}\overline{\lambda}_n(z-\lambda_n)/(\lambda_n(z-\overline{\lambda}_n))$. Let $\mathcal{K}=\mathcal{H}\ominus[\mathcal{D}_+\oplus\mathcal{D}_-]$. Denote by $F(\lambda,r)$ a solution y of $y''(r)=-\lambda^2\rho(r)y(r)$ on $[0,\infty)$ satisfying $y(0)=0$ and $y(r)=e^{i\lambda(a-r)}$ for $a\le r$. The main result states that if $g=(g_0,g_1)\in\mathcal{K}$ then $G(\lambda)=\langle F(\lambda,\cdot),\,g\rangle_{\mathcal{H}}/\sqrt{2\pi}$ belongs to the space $K=H^2_+\oplus S(\lambda)H^2_+$ and can be reconstructed from its values at the points $\{\lambda_n\}_{n\in\mathbb{Z}}$ by means of the interpolation

$$G(\lambda)=\sum_{n=-\infty}^{\infty}G(\lambda_n)\frac{P(\lambda)}{(\lambda-\lambda_n)P'(\lambda_n)}$$

where

$$P(\lambda)=\frac{1}{2}\Big(\frac{1}{i\lambda}\Big[\frac{d}{dx}\Psi_-(\lambda,x)\Big|_{x=a}\Big]+\Psi_-(\lambda,a)\Big).$$

Here $\Psi_-(\lambda,x)=F(\lambda,x)-F(-\lambda,x)M(\lambda)/M(-\lambda)$ where $M(\lambda)=\lim_{r\to0}F(\lambda,r)$. The space K can be viewed as the transformational image of a space of functions that are supported in an interval just as the Paley-Wiener spaces are images of functions supported in a fixed interval under the Fourier transform. In this sense, K is a space of bandlimited functions. *Joseph Lakey (Las Cruces)*

Keywords: Shannon sampling; Hardy space; bandlimited function; Regge problem; Lax-Phillips scattering theory
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