
io-port 05268610**Burke, J.V.; Lewis, A.S.; Overton, M.L.****Convexity and Lipschitz behavior of small pseudospectra.**

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Consider the space \mathbb{M}^n of all $n \times n$ complex matrices together with operator 2-norm $\|\cdot\|$. We denote the spectrum of a matrix $A \in \mathbb{M}^n$ by $\Lambda(A)$. For each $\varepsilon > 0$, the ε -pseudospectrum $\Lambda_\varepsilon(A)$ of A is the union of the spectra $\Lambda(X)$ for all X with $\|A - X\| \leq \varepsilon$. It is known that this pseudospectrum can be described as a finite union of sets each defined using finitely many polynomial inequalities; furthermore, each connected component of the pseudospectrum is compact and contains an eigenvalue of A , so, there are at most n components [see *R. Benedetti* and *J.-J. Risler*, *Real Algebraic and Semi-Algebraic Sets*, Hermann, Paris (1990; Zbl 0694.14006)]. In general, a component of the pseudospectrum need not be convex. The main theorem of the present paper is the following. Suppose that λ is a nonderogatory eigenvalue of a matrix A_0 in \mathbb{M}^n . For any sufficiently small $\mu > 0$, there exists $\bar{\varepsilon}$ with $0 < \bar{\varepsilon} < \mu$ such that for all ε with $0 < \varepsilon < \bar{\varepsilon}$ and all matrices A in some neighbourhood of A_0 (depending on μ and ε), the set

$$\tilde{\Lambda}_\varepsilon(A) := \{z \in \Lambda_\varepsilon(A) : |z - \lambda| < \mu\}$$

has the following properties: (i) $\tilde{\Lambda}_\varepsilon(A)$ is the component of $\Lambda_\varepsilon(A)$ containing λ and it contains no other eigenvalue of A_0 ; (ii) $\tilde{\Lambda}_\varepsilon(A)$ is compact and strictly convex; (iii) the set value mapping $\tilde{\Lambda}_\varepsilon$ is Lipschitz on a neighbourhood of A_0 with respect to the Hausdorff distance. As a corollary it follows that, if A_0 is nonderogatory, then for all sufficiently small $\varepsilon > 0$ the dependence of the pseudospectrum $\Lambda_\varepsilon(A)$ on the matrix A is Lipschitz with respect to the Hausdorff measure for A near A_0 . In a note added in proof, the authors point out that this corollary has since been generalized in a recent preprint of *A.S. Lewis* and *C.H.J. Pang* [SIAM J. Optim. 19, No.3, 1048-1072 (2008)].

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