

io-port 05317261**Dragan, Feodor F.; Fomin, Fedor V.; Golovach, Petr A.****Spanners in sparse graphs.**

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Summary: A t -spanner of a graph G is a spanning subgraph S in which the distance between every pair of vertices is at most t times their distance in G . If S is required to be a tree then S is called a tree t -spanner of G . In 1998, Fekete and Kremer showed that on unweighted planar graphs the tree t -spanner problem (the problem to decide whether G admits a tree t -spanner) is polynomial time solvable for $t \leq 3$ and is NP-complete as long as t is part of the input. They also left as an open problem whether the tree t -spanner problem is polynomial time solvable for every fixed $t \geq 4$. In this work we resolve this open problem and extend the solution in several directions. We show that for every fixed t , it is possible in polynomial time not only to decide if a planar graph G has a tree t -spanner, but also to decide if G has a t -spanner of bounded treewidth. Moreover, for every fixed values of t and k , the problem, for a given planar graph G to decide if G has a t -spanner of treewidth at most k , is not only polynomial time solvable, but is fixed parameter tractable (with k and t being the parameters). In particular, the running time of our algorithm is linear with respect to the size of G . We extend this result from planar to a much more general class of sparse graphs containing graphs of bounded genus. An apex graph is a graph obtained from a planar graph G by adding a vertex and making it adjacent to some vertices of G . We show that the problem of finding a t -spanner of treewidth k is fixed parameter tractable on graphs that do not contain some fixed apex graph as a minor, i.e. on apex-minor-free graphs. Graphs of bounded treewidth are sparse graphs and our technique can be used to settle the complexity of the parameterized version of the sparse t -spanner problem, where for given t and m one asks if a given n -vertex graph has a t -spanner with at most $n - 1 + m$ edges. Our results imply that the sparse t -spanner problem is fixed parameter tractable on apex-minor-free graphs with t and m being the parameters. Finally we show that the tractability border of the t -spanner problem cannot be extended beyond the class of apex-minor-free graphs. In particular, we prove that for every $t \geq 4$, the problem of finding a tree t -spanner is NP-complete on K_6 -minor-free graphs. Thus our results are tight, in a sense that the restriction of input graph being apex-minor-free cannot be replaced by H -minor-free for some non-apex fixed graph H .

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