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Erdős-Hajnal-type theorems in hypergraphs.

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Summary: The Erdős-Hajnal conjecture states that if a graph on n vertices is H-free, that is, it does not contain an induced copy of a given graph H, then it must contain either a clique or an independent set of size $n^{\delta(H)}$, where $\delta(H) > 0$ depends only on the graph H. Except for a few special cases, this conjecture remains wide open. However, it is known that an H-free graph must contain a complete or empty bipartite graph with parts of polynomial size. We prove an analogue of this result for 3-uniform hypergraphs, showing that if a 3-uniform hypergraph on n vertices is $\mathcal{H}$-free, for any given $\mathcal{H}$, then it must contain a complete or empty tripartite subgraph with parts of order $c(\log n)^{\frac{1}{2}+\delta(\mathcal{H})}$, where $\delta(\mathcal{H}) > 0$ depends only on $\mathcal{H}$. This improves on the bound of $c(\log n)^{\frac{1}{2}}$, which holds in all 3-uniform hypergraphs, and, up to the value of the constant $\delta(\mathcal{H})$, is best possible. We also prove that, for $k \geq 4$, no analogue of the standard Erdős-Hajnal conjecture can hold in k-uniform hypergraphs. That is, there are k-uniform hypergraphs $\mathcal{H}$ and sequences of $\mathcal{H}$-free hypergraphs which do not contain cliques or independent sets of size appreciably larger than one would normally expect.

Keywords: Ramsey theory; Erdős-Hajnal conjecture; hypergraphs
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