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Weighted Harary indices of apex trees and $k$-apex trees.


Summary: If $G$ is a connected graph, then $H_A(G) = \sum_{u \neq v} (\deg(u) + \deg(v))/d(u,v)$ is the additively Harary index and $H_M(G) = \sum_{u \neq v} \deg(u) \deg(v)/d(u,v)$ the multiplicatively Harary index of $G$. $G$ is an apex tree if it contains a vertex $x$ such that $G - x$ is a tree and is a $k$-apex tree if $k$ is the smallest integer for which there exists a $k$-set $X \subseteq V(G)$ such that $G - X$ is a tree. Upper and lower bounds on $H_A$ and $H_M$ are determined for apex trees and $k$-apex trees. The corresponding extremal graphs are also characterized in all the cases except for the minimum $k$-apex trees, $k \geq 3$. In particular, if $k \geq 2$ and $n \geq 6$, then $H_A(G) \leq (k + 1)(3n^2 - 5n - k^2 - k + 2)/2$ holds for any $k$-apex tree $G$, equality holding if and only if $G$ is the join of $K_k$ and $K_{1,n-k-1}$.

Keywords: additively Harary index; multiplicatively Harary index; apex tree; $k$-apex tree; harmonic number
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