Expected computations on color spanning sets.

Summary: Given a set of \( n \) points, each is painted by one of the \( k \) given colors, we want to choose \( k \) points with distinct colors to form a color spanning set. For each color spanning set, we can construct the convex hull and the smallest axis-aligned enclosing rectangle, etc. Assuming that each point is chosen independently and identically from the subset of points of the same color, we propose an \( O(n^2) \) time algorithm to compute the expected area of convex hulls of the color spanning sets and an \( O(n^2) \) time algorithm to compute the expected perimeter of convex hulls of the color spanning sets. For the expected perimeter (resp. area) of the smallest perimeter (resp. area) axis-aligned enclosing rectangles of the color spanning sets, we present an \( O(n \log n) \) (resp. \( O(n^2) \)) time algorithm. We also propose a simple approximation algorithm to compute the expected diameter of the color spanning sets. For the expected distance of the closest pair, we show that it is \(#P\)-complete to compute and there exists no polynomial time \( 2^{n^{1-\epsilon}} \) approximation algorithm to compute the probability that the closest pair distance of all color spanning sets equals to a given value \( d \) unless \( P = NP \), even in one dimension and when each color paints two points.

Keywords: expected value; imprecise data; computational geometry

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