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Enumeration of Golomb rulers and acyclic orientations of mixed graphs.

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Summary: A Golomb ruler is a sequence of distinct integers (the markings of the ruler) whose pairwise differences are distinct. Golomb rulers, also known as Sidon sets and B_2 sets, can be traced back to additive number theory in the 1930s and have attracted recent research activities on existence problems, such as the search for optimal Golomb rulers (those of minimal length given a fixed number of markings). Our goal is to enumerate Golomb rulers in a systematic way: we study

$$g_m(t) := \#\left\{\mathbf{x} \in \mathbb{Z}^{m+1} : 0 = x_0 < x_1 < \cdots < x_m = t, \text{ all } x_j - x_k \text{ distinct}\right\},$$

the number of Golomb rulers with $m+1$ markings and length t.Our main result is that $g_m(t)$ is a quasipolynomial in t which satisfies a combinatorial reciprocity theorem: $(-1)^{m-1}g_m(-t)$ equals the number of rulers $\mathbf{x}$ of length t with $m+1$ markings, each counted with its Golomb multiplicity, which measures how many combinatorially different Golomb rulers are in a small neighborhood of $\mathbf{x}$. Our reciprocity theorem can be interpreted in terms of certain mixed graphs associated to Golomb rulers; in this language, it is reminiscent of Stanley's reciprocity theorem for chromatic polynomials. Thus in the second part of the paper we develop an analogue of Stanley's theorem to mixed graphs, which connects their chromatic polynomials to acyclic orientations.

Keywords: Golomb ruler; Sidon set; combinatorial reciprocity theorem; lattice point; inside-out polytope; Ehrhart quasipolynomial; mixed graph; proper coloring; acyclic orientation
http://www.combinatorics.org/ojs/index.php/eljc/article/view/v19i3p42