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The $\mathfrak{sl}_3$ Jones polynomial of the trefoil: a case study of q-holonomic sequences.

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A q-holonomic sequence is a sequence of functions satisfying a linear recursion relation. The key property of such sequences is that they are uniquely defined by their holonomic annihilating ideals plus finitely many initial values, so that they are completely determined by a finite amount of information. The importance of q-holonomic sequences as an "algorithmically nice" class of sequences was pointed out by *D. Zeilberger* [J. Comput. Appl. Math. 32, No. 3, 321–368 (1990; Zbl 0738.33001)], who used them to generate special functions identities. Abstractly, we know that coloured Jones polynomials are q–holonomic sequences by *S. Garoufalidis* and *T. T. Q. Lê* [Geom. Topol. 9, 1253–1293 (2005; Zbl 1078.57012)]. But, until this paper, nobody had ever computed any coloured Jones polynomial for any Lie algebra of rank greater than one. In particular, the AJ conjecture had never been tested. In this paper, using the "algorithmic niceness" of q-holonomic sequences, the authors determine by computer the $\mathfrak{sl}_3$ Jones polynomial of the trefoil knot, confirming the $\mathfrak{sl}_3$ AJ conjecture for this example. Theory ought to be tested against experiment, and having one explicit $\mathfrak{sl}_3$ Jones polynomial in our hands is exciting. It is the nature of the heart to want, and the reviewer hopes that many more examples of coloured Jones polynomials of various knots and links will now be calculated using the techniques of this paper. *Daniel Moskovich (Singapore)*

Keywords: colored Jones polynomial; knots; trefoil; torus knots; sl_3; rank 2 Lie algebras; q-holonomic sequence; recursion ideal; quantum topology; q-Weyl algebra; Groebner bases
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