
io-port 05961728**Ku, Cheng Yeaw; Wong, Kok Bin****Generalized D -graphs for nonzero roots of the matching polynomial.**

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Summary: Recently, *D. Bauer, H. J. Broersma, A. Morgana and E. Schmeichel* [J. Graph Theory 55, No. 4, 343–358 (2007; Zbl 1120.05067)] introduced a graph operator $D(G)$, called the D -graph of G , which has been useful in investigating the structural aspects of maximal Tutte sets in G with a perfect matching. Among other results, they proved a characterization of maximal Tutte sets in terms of maximal independent sets in the graph $D(G)$ and maximal extreme sets in G . This was later extended to graphs without perfect matchings by *A. Busch, M. Ferrara and N. Kahl* [Discrete Appl. Math. 155, No. 18, 2487–2495 (2007; Zbl 05216557)]. Let θ be a real number and $\mu(G, x)$ be the matching polynomial of a graph G . Let $\text{mult}(\theta, G)$ be the multiplicity of θ as a root of $\mu(G, x)$. We observe that the notion of D -graph is implicitly related to $\theta = 0$. In this paper, we give a natural generalization of the D -graph of G for any real number θ , and denote this new operator by $D_\theta(G)$, so that $D_\theta(G)$ coincides with $D(G)$ when $\theta = 0$. We prove a characterization of maximal θ -Tutte sets which are θ -analogues of maximal Tutte sets in G . In particular, we show that for any $X \subseteq V(G)$, $|X| > 1$, and any real number θ , $\text{mult}(\theta, G \setminus X) = \text{mult}(\theta, G) + |X|$ if and only if $\text{mult}(\theta, G \setminus uv) = \text{mult}(\theta, G) + 2$ for any $u, v \in X$, $u \neq v$, thus extending the preceding work of *D. Bauer et al.* [loc. cit.] and *A. Busch et al.* [loc. cit.] which established the result for the case $\theta = 0$. Subsequently, we show that every maximal θ -Tutte set X is matchable to an independent set Y in G ; moreover, $D_\theta(G)$ always contains an isomorphic copy of the subgraph induced by $X \cup Y$. To this end, we introduce another related graph $S_\theta(G)$ which is a supergraph of G , and prove that $S_\theta(G)$ and G have the same Gallai-Edmonds decomposition with respect to θ . Moreover, we determine the structure of $D_\theta(G)$ in terms of its Gallai-Edmonds decomposition and prove that $D_\theta(S_\theta(G)) = D_\theta(G)$.

Keywords: matching polynomial; Gallai-Edmonds decomposition; Tutte sets; extreme sets; D -graphs
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