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**Cohen, Albert; DeVore, Ronald; Nochetto, Ricardo H.**

**Convergence rates of AFEM with $H^{-1}$ data.**

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Summary: This paper studies adaptive finite element methods (AFEMs), based on piecewise linear elements and newest vertex bisection, for solving second order elliptic equations with piecewise constant coefficients on a polygonal domain $\Omega \subset \mathbb{R}^2$. The main contribution is to build algorithms that hold for a general right-hand side $f \in H^{-1}(\Omega)$. Prior work assumes almost exclusively that $f \in L^2(\Omega)$. New data indicators based on local $H^{-1}$ norms are introduced, and then the AFEMs are based on a standard bulk chasing strategy (or Dörfler marking) combined with a procedure that adapts the mesh to reduce these new indicators. An analysis of our AFEM is given which establishes a contraction property and optimal convergence rates $N^{-s}$ with $0 < s \leq 1/2$. In contrast to previous work, it is shown that it is not necessary to assume a compatible decay $s < 1/2$ of the data estimator, but rather that this is automatically guaranteed by the approximability assumptions on the solution by adaptive meshes, without further assumptions on $f$; the borderline case $s = 1/2$ yields an additional factor $\log N$. Computable surrogates for the data indicators are introduced and shown to also yield optimal convergence rates $N^{-s}$ with $s \leq 1/2$.

*Keywords:* adaptivity; $H^{-1}$ data; contraction; rates; cardinality; approximation classes
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