A new computational method for the sparsest solutions to systems of linear equations.

Summary: The connection between the sparsest solution to an underdetermined system of linear equations and the weighted $\ell_1$-minimization problem is established in this paper. We show that seeking the sparsest solution to a linear system can be transformed to searching for the densest slack variable of the dual problem of weighted $\ell_1$-minimization with all possible choices of nonnegative weights. Motivated by this fact, a new reweighted $\ell_1$-algorithm for the sparsest solutions of linear systems, going beyond the framework of existing sparsity-seeking methods, is proposed in this paper. Unlike existing reweighted $\ell_1$-methods that are based on the weights defined directly in terms of iterates, the new algorithm computes a weight in dual space via certain convex optimization and uses such a weight to locate the sparsest solutions. It turns out that the new algorithm converges to the sparsest solutions of linear systems under some mild conditions that do not require the uniqueness of the sparsest solutions. Empirical results demonstrate that this new computational method remarkably outperforms $\ell_1$-minimization and stands as one of the very efficient sparsity-seeking algorithms for the sparsest solutions of systems of linear equations.

Keywords: $\ell_0$-minimization; sparsest solution; reweighted $\ell_1$-method; convex optimization; linear programming; bilevel optimization; sparsity recovery

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