Interpretability degrees of finitely axiomatized sequential theories.

Visser, Albert


Before explaining Albert Visser’s main achievement in his article under review, we need to introduce some definitions and notation. – A finitely axiomatized theory is a theory presented to us by giving a finite list of axioms. – The adjunctive set theory AS is the theory with a binary relation ∈, the axiom ∃x ∀y ¬y ∈ x saying that there is the empty set, and the axiom ∀u,v ∃x ∀y (y ∈ x ↔ y ∈ u ∨ y = v) saying that, given sets u and v, there is the set u ∪ {v}. – A direct interpretation is an interpretation that is unrelativized (has the trivial domain) and identity-preserving (translates identity to identity). – A sequential theory is a theory that directly interprets AS. – We fix a finitely axiomatized theory A that interprets Robison’s arithmetic Q. – We denote by V′ A the set of all finite extensions of A in the same language as A. – We denote by <′ A the binary relation on V′ A defined by: U <′ A V if and only if V interprets U. – We denote by ≡ A the equivalence relation on V′ A defined by: U ≡ A V if and only if U <′ A V and V <′ A U. – We denote by ∨ A the quotient V′ A/≡ A and call its elements degrees of interpretability. – We denote by < A the binary relation on ∨ A induced by <′ A. Now that we introduced the definitions and notation, we can explain Albert Visser’s main achievement in his article. Vítězslav Švejdar asked if (∨ A, < A) is a lattice. Albert Visser answers affirmatively when A is a sequential theory. He achieves this answer by proving a stronger convexity result: If A is sequential, B is a sequential and finitely axiomatized theory, and A < A B, then there is a finitely axiomatized extension C of A in the same language as A such that B ≡ C.

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