Vectorial variational principle with variable set-valued perturbation.

Summary: We give a general vectorial Ekeland’s variational principle, where the objective function is defined on an F-type topological space and taking values in a pre-ordered real linear space. Being quite different from the previous versions of vectorial Ekeland’s variational principle, the perturbation in our version is no longer only dependent on a fixed positive vector or a fixed family of positive vectors. It contains a family of set-valued functions taking values in the positive cone and a family of subadditive functions of topology generating quasi-metrics. Hence, the direction of the perturbation in the new version is a family of variable subsets which are dependent on the objective function values. The general version includes and improves a number of known versions of the vectorial Ekeland’s variational principle. From the general Ekeland’s principle, we deduce the corresponding versions of Caristi-Kirk’s fixed point theorem and Takahashi’s nonconvex minimization theorem. Finally, we prove that all the three theorems are equivalent to each other.

Keywords: vectorial Ekeland’s variational principle; F-type topological space; locally convex space; pre-ordered linear space; direction of perturbation

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