Downhill domination problem in graphs.

Summary: A path $\pi = (v_1, v_2, \ldots, v_{k+1})$ in a graph $G = (V, E)$ is a downhill path if for every $i$, $1 \leq i \leq k$, $\deg(v_i) \geq \deg(v_{i+1})$, where $\deg(v_i)$ denotes the degree of vertex $v_i \in V$. A downhill dominating set DDS is a set $S \subseteq V$ having the property that every vertex $v \in V$ lies on a downhill path originating from some vertex in $S$. The downhill domination number $\gamma_{dn}(G)$ equals the minimum cardinality of a DDS of $G$. A DDS having minimum cardinality is called a $\gamma_{dn}$-set of $G$. In this note, we give an enumeration of all the distinct $\gamma_{dn}$-sets of $G$, and we show that there is a linear time algorithm to determine the downhill domination number of a graph.

Keywords: combinatorial problems; downhill domination number; direct dominating set; linear time algorithm

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