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Mesh algorithms for solving principal Diophantine equations, sand-glass tubes and tori of roots.

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Summary: We study integral solutions of diophantine equations $q(x) = d$, where $x = (x_1, \ldots, x_n), n \geq 1, d \in \mathbb{Z}$ is an integer and $q : \mathbb{Z}^n \to \mathbb{Z}$ is a non-negative homogeneous quadratic form. Contrary to the negative solution of the Hilbert's tenth problem, for any such a form $q(x)$, we give efficient algorithms describing the set $\mathcal{R}_q(d)$ of all integral solutions of the equation $q(x) = d$ in a Φ_A-mesh translation quiver form. We show in Section 5 that usually the set $\mathcal{R}_q(d)$ has a shape of a Φ_A-mesh sand-glass tube or of a Φ_A-mesh torus, see 5.8, 5.10, and 5.13. If, in addition, the subgroup Ker $q = \{v \in \mathbb{Z}^n; q(v) = 0\}$ of $\mathbb{Z}^n$ is infinite cyclic, we study the solutions of the equations $q(x) = 1$ by applying a defect $\delta A : \mathbb{Z}^n \to \mathbb{Z}$ and a reduced Coxeter number $\check{c}_A \in \mathbb{N}$ defined by means of a morsification $b_A : \mathbb{Z}^n \times \mathbb{Z}^n \to \mathbb{Z}$ of q, see Section 4. On this way we get a simple graphical algorithm that constructs all integral solutions in the shape of a mesh translation oriented graph consisting of Coxeter Φ_A-orbits. It turns out that usually the graph has at most three infinite connected components and each of them has an infinite band shape, or an infinite horizontal tube shape, or has a sand-glass tube shape. The results have important applications in representation theory of groups, algebras, quivers and partially ordered sets, as well as in the study of derived categories (in the sense of Verdier) of module categories and categories of coherent sheaves over algebraic varieties.

Keywords: principal quadratic form; quiver; poset; tubular mesh algorithm; Coxeter matrix; morsification; defect; reduced Coxeter number; sand-glass tube; mesh geometry
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